Part II
Supplement for "Optimal Convergence Rates, Bahadur Representation, and Asymptotic Normality of Partitioning Estimators"

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March 12, 2013

Abstract

This is a supplemental appendix for "Optimal Convergence Rates, Bahadur Representation, and Asymptotic Normality of Partitioning Estimators". We first present detailed proofs of all the theoretical results from the main text. Lemmas and other claims are restated before proof, and as such this section may replace the appendix contained in the main manuscript. Note that equation numbers may change. Next, for the special case of the piecewise constant partitioning estimator we characterize the leading terms of an *unconditional* integrated mean-square error expansion. The result agrees with Theorem 3, specialized to the same case. Finally, numerous additional simulation results are presented, vastly expanding the discussion in Section 5.

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A Proof of Theorems

Let C denote a generic positive constant that may take different values in different places. If specific constants are needed they will be numbered consecutively. For scalars, vectors, or matrixes, let $|\cdot|$ be the Euclidean norm. To denote the various products, we use $\times$ for Cartesian, $\bigotimes$ for Kronecker, and $\prod$ for usual multiplication over arguments: any may be repeated, as in $\times_{\ell=1}^{d}$. Any use of the "times" symbol $\times$ will be clear from the context; examples include matrix dimensions and line breaks in displayed equations. Matrix inequalities are understood to be in the positive definite sense. Consecutive uses of the symbol $\asymp$ are to be interpreted pairwise. All results below hold for the partitioning schemes described in the text. For a generic cell P_j, let p_{j*}, $\overline{p}_j$, and p_j^* be the vectors in $\mathbb{R}^d$ giving the start, mid-point, and end of the cell, respectively, where the start and end are defined in distance to the origin. For a multi-index k, we define the additional notation: $k! = k_1! \cdots k_d!$, $k \le \tilde{k} \Leftrightarrow k_1 \le \tilde{k}_1, \ldots, k_d \le \tilde{k}_d$, and $\sum_{[k]\le K} = \sum_{L=0}^{K} \sum_{[k]=L}$ for $K \ge 0$.

Prior to proving the main results, it is convenient to take a nonsingular linear transformation of the polynomial basis. The estimator $\hat{\mu}(x)$ is invariant to such rotations, thus without loss of generality we may take the basis to be centered at the midpoint of each cell and scaled by the length of the cell. Observe that centering the polynomial basis around the center of each cell avoids issues of differentiability at the boundary of each cell and the support $\mathcal{X}$. Recall that $R(x)$ is ordered ascendingly in $k \in \mathbb{Z}_+^d$ and $\ell = 1, \ldots, d$. Define the one-to-one function $g(k) : \mathbb{Z}_+^d \to \mathbb{N}$ that gives the index position of $R(x)$ corresponding to entry x^k. Let $g^* = \max_k\{g(k) : k \in \mathbb{Z}_+^d,\ [k] \le K-1\}$. Then $R(x)$ is a $g^* \times 1$ vector with element $g(k)$ equal to x^k for all $\{k \in \mathbb{Z}_+^d : [k] \le K-1\}$. As $R(x)$ excludes terms with degree exceeding $K-1$, it follows that $g^* \le K^d$. To fix ideas, consider the two simple cases from the text: if $K = 1$, then (for any d) $R(x) = (1)$ and hence $K^d = g^* = 1$; if $K = 2$ and $d = 2$ say, then $R(x) = (1, x_1, x_2)'$ and $K^d = 4,\ g^* = 3$.

Recall from the text that the interval endpoints $p_{\ell,j-1}$ and $p_{\ell,j}$, for $j = 1, \ldots, J_n$, define the partition of the ℓ-dimension of $\mathcal{X}$, and let $\overline{p}_{\ell,j} = (p_{\ell,j} + p_{\ell,j-1})/2 \in \mathbb{R}$ be the midpoint of each interval. Define the matrix functions $D(a)$ to be the $K \times K$ diagonal matrix with entries given by $a^{-(v-1)}$, $v = 1, \ldots, K$ and $L(b)$ to be the $K \times K$ lower triangular matrix with typical element $\binom{u-1}{v-1}(-b)^{u-v}$, $(u, v) \in \{1, \ldots, K : u \ge v\}$. We then take the (rotated) polynomial basis to be given by

$$\tilde{R}_j(x) \equiv \mathbb{1}_{P_j}(x)\tilde{R}(x) = \mathbb{1}_{P_j}(x) S_K \bigotimes_{\ell=1}^{d} \left\{ D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right) L(\overline{p}_{\ell,j}) r(x_\ell) \right\}.$$

Each element of the product $L(\overline{p}_{\ell,j}) r(x_\ell)$ is (the binomial expansion of) $(x_\ell - \overline{p}_{\ell,j})^{k_\ell}$, $0 \le k_\ell \le K-1$, and premultiplication by $D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right)$ rescales appropriately. To be explicit, for the ℓ-dimension, with $\underline{p}_{\ell,j} = 1/(p_{\ell,j} - \overline{p}_{\ell,j})$ the product $D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right) L(\overline{p}_{\ell,j}) r(x_\ell)$ is given by:

$$\begin{pmatrix} 1 & & & & \\ & \underline{p}_{\ell,j} & & & \\ & & \underline{p}_{\ell,j}^2 & & \\ & & & \ddots & \\ & & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} 1 & & & & \\ -\overline{p}_{\ell,j} & 1 & & & \\ \overline{p}_{\ell,j}^2 & -2\overline{p}_{\ell,j} & 1 & & \\ \vdots & & & & \\ (-\overline{p}_{\ell,j})^{K-1} & \cdots\cdots\cdots & & -(K-1)\overline{p}_{\ell,j} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x_r \\ x_r^2 \\ \vdots \\ x_r^{K-1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & & \\ & \underline{p}_{\ell,j} & & & \\ & & \underline{p}_{\ell,j}^2 & & \\ & & & \ddots & \\ & & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} \sum_{k_\ell=0}^{0} \binom{0}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{0-k_\ell} \\ \sum_{k_\ell=0}^{1} \binom{1}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{1-k_\ell} \\ \sum_{k_\ell=0}^{2} \binom{2}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{2-k_\ell} \\ \vdots \\ \sum_{k_\ell=0}^{K-1} \binom{K}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{K-1-k_\ell} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & & \\ & \underline{p}_{\ell,j} & & & \\ & & \underline{p}_{\ell,j}^2 & & \\ & & & \ddots & \\ & & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} 1 \\ (x_r - \overline{p}_{\ell,j}) \\ (x_r - \overline{p}_{\ell,j})^2 \\ \vdots \\ (x_r - \overline{p}_{\ell,j})^{K-1} \end{pmatrix} = r\left(\frac{x_\ell - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}}\right).$$

The matrix S_K is a $g^* \times K^d$ selection matrix which removes terms of degree exceeding $K-1$, with the properties that

$$S_K S_K' = I_{g^*} \quad \text{and} \quad S_K' S_K = \begin{bmatrix} I_{g^*} & Z_{g^*, K^d - g^*} \\ Z_{K^d - g^*, g^*} & Z_{K^d - g^*, K^d - g^*} \end{bmatrix},$$

where Z_{z_1,z_2} is a $z_1 \times z_2$ matrix of zeros.

To see that this is equivalent to a transformation of the full basis, define $T_{\ell,j} = D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right) L(\overline{p}_{\ell,j})$ and observe that

$$\begin{aligned} \left[S_K \left(T_{1,j} \otimes \cdots \otimes T_{d,j}\right) S_K'\right] \tilde{R}_j(x) &= \mathbb{1}_{P_j}(x) S_K \left(T_{1,j} \otimes \cdots \otimes T_{d,j}\right) S_K' S_K \left(r(x_1) \otimes \cdots \otimes r(x_d)\right) \\ &= \mathbb{1}_{P_j}(x) S_K \left(T_{1,j} r(x_1) \otimes \cdots \otimes T_{d,j} r(x_d)\right), \end{aligned}$$

as required, relying upon $R(x)$ being ordered ascendingly in $k \in \mathbb{Z}_+^d$.

Finally let $\tilde{R}_j = (\tilde{R}_j(X_1), \ldots, \tilde{R}_j(X_n))'$ and (globally) redefine $\Omega_j = \mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right]/q_j$ and $\hat{\Omega}_j = \tilde{R}_j'\tilde{R}_j/(nq_j)$, maintaining the same notation for the latter two for simplicity.

A.1 Preliminary Lemmas

Several intermediate lemmas are required before proving the main results. These lemmas establish properties of partitioning estimators which may be of independent interest for other applications.

Lemma A.1. *Under Assumption 1(b), for $s \leq K-1$ the polynomial basis satisfies:*

$$\max_{1\leq j\leq J_n^d} \max_{m:[m]\leq s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_\infty = O\left(J_n^s\right).$$

Proof. The proof below can be read for either $d=1$ or $d>1$. The difference notational: in the $d=1$ case the indexes k and m are treated as simple integers, rather than multi-indexes (and $[k]$ is read as simply k), whereas for the $d>1$ case the multi-index notation is maintained. Recall the definitions of the vectors $\overline{p}_j$ and p_j^* above. By construction of the partition, for $x \in P_j$, $\left|x-\overline{p}_j\right| \leq |p_j^* - \overline{p}_j| \asymp J_n^{-1}$. Following rotation, each element of the basis is of the form $\frac{(x-\overline{p}_j)^k}{(p_j^*-\overline{p}_j)^k}$ for some $k \in \mathbb{Z}_+^d$. Hence for fixed $x \in \mathcal{X}$ and a multi-index m such that $[m] \leq K-1$, the norm (squared) of $\partial^m \tilde{R}_j(x)$ is the sum of squares over all such elements with $[k] \leq K-1$, restricted to the cell P_j:

$$\begin{aligned}
\left|\partial^m \tilde{R}_j(x)\right|^2 &= \mathbb{1}_{P_j}(x) \sum_{[k]\leq K-1} \left\{ \partial^m \frac{(x-\overline{p}_j)^k}{(p_j^*-\overline{p}_j)^k} \right\}^2 \\
&= \mathbb{1}_{P_j}(x) \sum_{[k]\leq K-1} \mathbb{1}\{m \leq k\} \left\{ \frac{k!}{(k-m)!} \frac{\left(x-\overline{p}_j\right)^{k-m}}{(p_j^*-\overline{p}_j)^k} \right\}^2 \\
&= \left(\frac{1}{(p_j^*-\overline{p}_j)^m} \right)^2 \mathbb{1}_{P_j}(x) \sum_{[k]\leq K-1} \mathbb{1}\{m \leq k\} \left\{ \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^*-\overline{p}_j)^{k-m}} \right\}^2 \\
&\leq C \left(\frac{1}{(p_j^*-\overline{p}_j)^m} \right)^2 = O\left(J_n^{2[m]}\right),
\end{aligned}$$

uniformly in $1 \leq j \leq J_n^d$, $x \in P_j$, and $\{m : [m] \leq K-1\}$, and in particular for those satisfying $[m] \leq s \leq K-1$, for any such s. □

Lemma A.2. *Define $\mu_j(x) \equiv \mathbb{1}_{P_j}(x)\mu(x)$, and following the definition in Eqn.* (2)*, $\partial^m \mu_j(x) \equiv \mathbb{1}_{P_j}(x)\partial^m \mu(x)$. Under Assumptions 1(b) and 1(e), there is a non-random vector β_j^0, depending only on K and j, such that for $s \leq S \wedge (K-1)$:*

$$\max_{1\leq j\leq J_n^d} \max_{m:[m]\leq s} \left\| \partial^m \mu_j(\cdot) - \partial^m \tilde{R}_j(\cdot)'\beta_j^0 \right\|_\infty = O\left(J_n^{-((S+\alpha)\wedge K-s)}\right).$$

Proof. The proof consists of two steps. The first is to show that $\partial^m \mu_j(x)$ admits a Taylor series approximation with remainder of the appropriate order. Second, we show that β_j^0 may be constructed so that $\partial^m \tilde{R}_j(x)'\beta_j^0$ is that Taylor series. Critical to this is that differentiation only operates on the basis, not the non-random vector β_j^0. We first present complete details for $d = 1$, to keep the notation simple. The extension to higher dimensions follows along the same lines and a more terse proof is given.

Take $d = 1$. For the first step of the proof we derive the order of the remainder using the integral representation. By Assumption 1(e), $\partial^m \mu_j(x)$ admits a Taylor series approximation of order $S \wedge (K-1) - m$ (at least). To save notation, let $\tilde{\mathrm{s}} = S \wedge (K-1)$. For $x \in P_j$, the remainder is given by:

$$
\begin{aligned}
&\left|\partial^m \mu_j(x) - \sum_{k=0}^{\tilde{\mathrm{s}}-m} \frac{\partial^{k+m}\mu_j(\overline{p}_j)}{k!}(x-\overline{p}_j)^k\right| \\
&\quad = \left|\partial^m \mu_j(x) - \sum_{k=0}^{\tilde{\mathrm{s}}-m-1} \frac{\partial^{k+m}\mu_j(\overline{p}_j)}{k!}(x-\overline{p}_j)^k - \frac{\partial^{\tilde{\mathrm{s}}}\mu_j(\overline{p}_j)}{(\tilde{\mathrm{s}}-m)!}(x-\overline{p}_j)^{(\tilde{\mathrm{s}}-m)}\right| \\
&\quad = \left|\frac{1}{(\tilde{\mathrm{s}}-m-1)!}\int_{\overline{p}_j}^{x} [\partial^{\tilde{\mathrm{s}}}\mu_j(z)](x-z)^{(\tilde{\mathrm{s}}-m-1)}dz - \frac{\partial^{\tilde{\mathrm{s}}}\mu_j(\overline{p}_j)}{(\tilde{\mathrm{s}}-m)!}(x-\overline{p}_j)^{(\tilde{\mathrm{s}}-m)}\right| \\
&\quad = \left|\frac{1}{(\tilde{\mathrm{s}}-m-1)!}\int_{\overline{p}_j}^{x} [\partial^{\tilde{\mathrm{s}}}\mu_j(t)](x-z)^{(\tilde{\mathrm{s}}-m-1)}dz - \frac{1}{(\tilde{\mathrm{s}}-m-1)!}[\partial^{\tilde{\mathrm{s}}}\mu_j(\overline{p}_j)]\int_{\overline{p}_j}^{x}(x-z)^{(\tilde{\mathrm{s}}-m-1)}dz\right| \\
&\quad = \frac{1}{(\tilde{\mathrm{s}}-m-1)!}\left|\int_{\overline{p}_j}^{x}(x-z)^{(\tilde{\mathrm{s}}-m-1)}\left[\partial^{\tilde{\mathrm{s}}}\mu_j(z) - \partial^{\tilde{\mathrm{s}}}\mu_j(\overline{p}_j)\right]dz\right|.
\end{aligned}
$$

For notational purposes, define $\tilde{\alpha} = \alpha \mathbb{1}\{K \geq S+1\} + (1)\mathbb{1}\{K < S+1\}$. Under Assumption 1(e), there is a constant C_1 depending only on $\mu(\cdot)$ and $\tilde{\mathrm{s}}$ such that $\left|\partial^{\tilde{\mathrm{s}}}\mu_j(z) - \partial^{\tilde{\mathrm{s}}}\mu_j(\overline{p}_j)\right| \leq C_1 \left|z - \overline{p}_j\right|^{\tilde{\alpha}}$, for all j. The notation of $\tilde{\alpha}$ is introduced because if $K < S+1$, then $\partial^{\tilde{\mathrm{s}}}\mu_j(z)$ is Lipschitz continuous (i.e. Hölder continuous with $\alpha = 1$). Hence the above display is:

$$
\leq \frac{C_1}{(\tilde{\mathrm{s}}-m-1)!}\int_{\overline{p}_j}^{x} |x-z|^{(\tilde{\mathrm{s}}-m-1)}|z-\overline{p}_j|^{\tilde{\alpha}}dz,
$$

which by the construction of the partition and the range of integration is:

$$
\begin{aligned}
&\leq \frac{C_1}{(\tilde{\mathrm{s}}-m-1)!}\left|x-\overline{p}_j\right|^{\tilde{\mathrm{s}}-m+\tilde{\alpha}} \\
&\leq \frac{C_1}{(\tilde{\mathrm{s}}-m-1)!}\left(\frac{p_{J_n}^* - p_{1*}}{J_n}\right)^{\tilde{\mathrm{s}}-m+\tilde{\alpha}}.
\end{aligned}
$$

This bound is uniform in $x \in P_j$ and $1 \leq j \leq J_n$. The difference $(p_{J_n}^* - p_{1*})$ represents the length of the support $\mathcal{X}$, which under Assumption 1(b) is a bounded constant. Hence, as m appears only in the denominator and the exponent:

$$\max_{1\leq j\leq J_n^d}\max_{1\leq m\leq s}\left|\partial^m\mu_j(x)-\sum_{k=0}^{\tilde{s}-m}\frac{\partial^{k+m}\mu_j(\overline{p}_j)}{k!}(x-\overline{p}_j)^k\right|\leq\max_{1\leq j\leq J_n^d}\max_{1\leq m\leq s}\frac{C_1}{(\tilde{s}-m-1)!}\left(\frac{p^*_{J_n}-p_{1*}}{J_n}\right)^{\tilde{s}-m+\tilde{\alpha}}$$
$$=O\left(J_n^{-((S+\alpha)\wedge K-s)}\right).$$

To complete the proof for $d=1$, we now construct β^0_j such that $\partial^m\tilde{R}_j(x)'\beta^0_j$ is the Taylor series approximation for $\partial^m\mu(x)$. Differentiation operates only on the vector $\tilde{R}_j(x)$. The first $m-1$ entries of $\partial^m\tilde{R}_j(x)$ are zero. Thus, element k of $\partial^m\tilde{R}_j(x)$ is given by

$$\mathbb{1}\{m\leq k\}\frac{k!}{(k-m)!}\frac{(x-\overline{p}_j)^{k-m}}{(p^*_j-\overline{p}_j)^k}.$$

Define the coefficient vector β^0_j with entry k equal to

$$\frac{1}{k!}[\partial^k\mu_j(\overline{p}_j)](p^*_j-\overline{p}_j)^k.$$

Then we have

$$\partial^m\tilde{R}_j(x)'\beta^0_j=\sum_{k=0}^{S\wedge(K-1)}\mathbb{1}\{m\leq k\}\frac{k!}{(k-m)!}\frac{(x-\overline{p}_j)^{k-m}}{(p^*_j-\overline{p}_j)^k}\frac{1}{k!}[\partial^k\mu_j(\overline{p}_j)](p^*_j-\overline{p}_j)^k$$
$$=\sum_{k\geq m}^{S\wedge(K-1)}\frac{1}{(k-m)!}(x-\overline{p}_j)^{k-m}[\partial^k\mu_j(\overline{p}_j)],$$

and re-indexing the sum by changing variables using $\tilde{k}=k-m$ this is equal to

$$=\sum_{\tilde{k}=0}^{S\wedge(K-1)-m}\frac{\partial^{\tilde{k}+m}\mu_j(\overline{p}_j)}{\tilde{k}!}(x-\overline{p}_j)^{\tilde{k}}.$$

This expression now exactly matches the Taylor approximation of $\partial^m\mu_j(x)$ given above. This completes the proof for $d=1$.

Now consider any $d\geq 1$. Just as above, Assumption 1(e) implies that $\partial^m\mu_j(x)$ satisfies the Taylor expansion for $x\in P_j$ given by:

$$\partial^m\mu_j(x)=\sum_{[k]\leq S\wedge(K-1)-[m]}\frac{1}{k!}\left(\partial^{k+m}\mu_j(\overline{p}_j)\right)\left(x-\overline{p}_j\right)^k+O\left(\left|x-\overline{p}_j\right|^{(S+\alpha)\wedge K-[m]}\right),\qquad\text{(A.1)}$$

with constants which can be made uniform in the multi-index m, s, and j. The terms of the summation are assumed to be ordered ascendingly in $g(k)$ as defined above. It remains to construct β_j^0 appropriately so that $\partial^m \tilde{R}_j(x)'\beta_j^0$ is the Taylor approximation given as the first term on the right hand side of (A.1). Recall the multi-index notational conventions defined earlier. For fixed $m \in \mathbb{Z}_+^d, [m] \leq s$, any entry of $\partial^m \tilde{R}_j(x)$ with $k \leq m$ is zero. Thus, entry $g(k)$ of $\partial^m \tilde{R}_j(x)$ is given by:

$$\mathbb{1}\{m \leq k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k}.$$

Next, for $k \in \mathbb{Z}_+^d$ define the function $\beta_j^0(k)$ as:

$$\beta_j^0(k) = \frac{1}{k!}\left(\partial^k \mu_j(\overline{p}_j)\right)\left(p_j^* - \overline{p}_j\right)^k.$$

As $g(k)$ is one-to-one and returns the index position of the entry corresponding to multi-index k, we can define the coefficient vector β_j^0 as the $g^* \times 1$ vector with entry e equal to $\beta_j^0(g^{-1}(e))$, for all entries $e = 1, \ldots, g^*$, where we note that $g^{-1}(e) \in \mathbb{Z}_+^d$ is a multi-index valued function. Therefore:

$$\begin{aligned}
\partial^m \tilde{R}_j(x)'\beta_j^0 &= \sum_{[k] \leq S \wedge (K-1)} \mathbb{1}\{m \leq k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k} \frac{1}{k!}\left(\partial^k \mu_j(\overline{p}_j)\right)\left(p_j^* - \overline{p}_j\right)^k \\
&= \sum_{[k] \leq S \wedge (K-1)} \mathbb{1}\{m \leq k\} \frac{1}{(k-m)!}\left(x - \overline{p}_j\right)^{k-m} \partial^k \mu_j(\overline{p}_j).
\end{aligned}$$

By definition, the multi-index satisfies $[k + \tilde{k}] = [k] + [\tilde{k}]$, and so re-indexing the above sum by changing variables $\tilde{k} = k - m$, we obtain

$$\partial^m \tilde{R}_j(x)'\beta_j^0 = \sum_{[\tilde{k}+m] \leq S \wedge (K-1)} \frac{1}{\tilde{k}!}\left(\partial^{\tilde{k}+m} \mu_j(\overline{p}_j)\right)\left(x - \overline{p}_j\right)^{\tilde{k}}.$$

This matches the Taylor series, hence subtracting from Eqn. (A.1) gives:

$$\begin{aligned}
\max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \left\| \partial^m \mu_j(x) - \partial^m \tilde{R}_j(x)'\beta_j^0 \right\|_\infty &= O\left(\max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \sup_{x \in P_j} \left| x - \overline{p}_j \right|^{(S+\alpha) \wedge K - [m]} \right) \\
&= O\left(J_n^{-((S+\alpha) \wedge K - [s])} \right),
\end{aligned}$$

completing the proof. □

Lemma A.3. *Recall that* $q_j = \mathbb{P}[X \in P_j]$ *and* $\Omega_j = \mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right]/q_j$. *Under Assumption 1,* $\Omega_j \asymp I_{g^*}$*, the identity matrix, uniformly in* j.

Proof. By Assumption 1(d) and the construction of the partition, $q_j = \int_{P_j} f(x)dx \asymp C \int_{P_j} dx =$

$C\,\mathrm{vol}\,(P_j) \asymp J_n^{-d}$. Applying this result and Assumption 1(d) again, we have:

$$\Omega_j = \frac{1}{q_j}\int_{\mathcal{X}} \tilde{R}_j(x)\tilde{R}_j(x)' f(x)dx \asymp J_n^d \int_{\mathcal{X}} \tilde{R}_j(x)\tilde{R}_j(x)' f(x)dx \asymp J_n^d \int_{\mathcal{X}} \tilde{R}_j(x)\tilde{R}_j(x)'dx.$$

Now, by Assumption 1(b), properties of the Kronecker product, and the construction of the transformed basis,

$$\begin{aligned}
\Omega_j &\asymp J_n^d S_K \int_{P_j} \left\{ \bigotimes_{\ell=1}^{d} r\left(\frac{x_\ell - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}}\right) r\left(\frac{x_\ell - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}}\right)' \right\} dx S_K' \\
&\asymp J_n^d S_K \bigotimes_{\ell=1}^{d} \left\{ \int_{p_{\ell,j-1}}^{p_{\ell,j}} r\left(\frac{x_\ell - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}}\right) r\left(\frac{x_\ell - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}}\right)' dx_\ell \right\} S_K'.
\end{aligned}$$

A change of variables using $z = (x_\ell - \overline{p}_{\ell,j})/(p_{\ell,j} - \overline{p}_{\ell,j})$, followed by the fact that $|p_{\ell,j} - p_{\ell,j-1}| \asymp J_n^{-1}$ shows that:

$$\begin{aligned}
\Omega_j &\asymp J_n^d \left(\prod_{\ell=1}^{d} \left| p_{\ell,j} - \overline{p}_{\ell,j} \right| \right) S_K \left\{ \bigotimes_{\ell=1}^{d} \int_{-1}^{1} r(z)r(z)'dz \right\} S_K' \\
&\asymp S_K \left\{ \bigotimes_{\ell=1}^{d} \int_{-1}^{1} r(z)r(z)'dz \right\} S_K',
\end{aligned}$$

For the change of variables $t = (z+1)/2$ we have $\int_{-1}^{1} z^k dz = 2\int_0^1 (2t-1)^k dt$. Applying this change of variables to the entire basis is equivalent to the inversion of the centering and scaling performed by the matrixes $L(\cdot)$ and $D(\cdot)$ defined earlier. Therefore:

$$\int_{-1}^{1} r(z)r(z)'dz = \int_0^1 \left[D(2)L(-1)\right]^{-1} r(t)r(t)' \left[L(-1)D(2)\right]^{-1} dt = \left[D(2)L(-1)\right]^{-1} H \left[L(-1)D(2)\right]^{-1},$$

where H denotes the Hilbert matrix of order K, which is positive definite. Collecting these results, where consecutive uses of the symbol $\asymp$ are interpreted pairwise, we have:

$$\Omega_j \asymp S_K \left\{ \bigotimes_{\ell=1}^{d} \left[D(2)L(-1)\right]^{-1} H \left[L(-1)D(2)\right]^{-1} \right\} S_K' \asymp I_{g^*}. \qquad \square$$

Lemma A.4. *Let* $a_n = n^{-1}J_n^d \log(J_n^d)$, *and recall* $\hat{\Omega}_j = \tilde{R}_j'\tilde{R}_j/(nq_j)$. *Under Assumption 1 and the rate restriction of Theorem 1:* $\max_{1\le j\le J_n^d} |\hat{\Omega}_j - \Omega_j|^2 = O_p(a_n)$. *If, in addition,* $J_n^d \asymp (n/\log(n))^\gamma$, $\gamma \in (0,1)$, *the same is true almost surely.*

Proof. For $k, \tilde{k} \in \mathbb{Z}_+^d$ with $[k], [\tilde{k}] \le K-1$, let the $(g(k), g(\tilde{k}))$ element of $(\hat{\Omega}_j - \Omega_j)$ be denoted

$\sum_{i=1}^{n} W_{ij}(k,\tilde{k})/(nq_j)$, where

$$W_{ij}(k,\tilde{k}) = \left[\tilde{R}_j(X_i)\tilde{R}_j(X_i)'\right]_{g(k),g(\tilde{k})} - \left[\mathbb{E}\left[\tilde{R}_j(X_i)\tilde{R}_j(X_i)'\right]\right]_{g(k),g(\tilde{k})}.$$

By Lemma A.1 (taking $s=0$) and the triangle inequality, $\left|W_{ij}(k,\tilde{k})\right| \leq 2\|\tilde{R}_j(\cdot)\|_\infty^2 < C$ and $\mathbb{E}\left[W_{ij}(k,\tilde{k})^2\right] \leq C\|\tilde{R}_j(\cdot)\|_\infty^4 \mathbb{E}\left[\mathbb{1}_{P_j}(X)\right] \leq Cq_j$, for any $k,\tilde{k}$. Thus by Boole's inequality, K being fixed, Bernstein's inequality, and finally applying $q_j \asymp J_n^{-d}$ and canceling where possible:

$$\begin{aligned}
\mathbb{P}\left[\max_{1\leq j\leq J_n^d}\left|\hat{\Omega}_j - \Omega_j\right| > (a_n)^{1/2}\varepsilon\right] &\leq J_n^d \max_{1\leq j\leq J_n^d}\mathbb{P}\left[\left|\hat{\Omega}_j - \Omega_j\right| > (a_n)^{1/2}\varepsilon\right] \\
&\leq CJ_n^d \max_{1\leq j\leq J_n^d}\max_{[k],[\tilde{k}]\leq K-1}\mathbb{P}\left[\left|\sum_{i=1}^{n} W_{ij}(k,\tilde{k})\right| > q_j\sqrt{nJ_n^d\log(J_n^d)}\varepsilon\right] \\
&\leq CJ_n^d \max_{1\leq j\leq J_n^d}\max_{[k],[\tilde{k}]\leq K-1}\exp\left\{-C\frac{q_j^2 nJ_n^d\log(J_n^d)\varepsilon^2}{nq_j + q_j\sqrt{nJ_n^d\log(J_n^d)}\varepsilon}\right\} \\
&\leq C\exp\left\{\log(J_n^d)\left[1 - C\frac{\varepsilon^2}{1+\sqrt{a_n}\varepsilon}\right]\right\},
\end{aligned}$$

which is arbitrarily small for ε large enough by the rate restriction of Theorem 1.

When $J_n^d \asymp (n/\log(n))^\gamma$, we use the above bound to write:

$$\sum_{n=1}^{\infty}\mathbb{P}\left[\max_{1\leq j\leq J_n^d}\left|\hat{\Omega}_j - \Omega_j\right| > (a_n)^{1/2}\varepsilon\right] \leq \sum_{n=1}^{\infty} C\left(\frac{n}{\log(n)}\right)^{\gamma - C\gamma\varepsilon^2/(1+\sqrt{a_n}\varepsilon)} < \infty,$$

where summability is ensured by choosing ε large enough and $a_n \to 0$ by the rate restriction in Theorem 1. The conclusion follows by the Borel-Cantelli Lemma. □

Lemma A.5. *Let the conditions of Theorem 2 hold, and for ξ therein let $r_n^2 = n^{-1}J_n^{d(2-\xi)}\log(J_n^d)^\xi$. Then for $G = (\mu(X_1),\ldots,\mu(X_n))'$, we have $\max_{1\leq j\leq J_n^d}|\tilde{R}_j'(Y-G)/(nq_j)| = O_p(r_n)$. If, in addition, $J_n^d \asymp (n/\log(n))^\gamma$, $\gamma\in(0,1)$, the same is true almost surely.*

Proof. With the convention $0/0=0$, define $t_n = J_n^{d\xi/\eta}\log(J_n^d)^{-\xi/\eta}$. Following the same notation as in Lemma A.4, let

$$\begin{aligned}
H_{ij}(k) &= \mathbb{1}_{P_j}(X_i)\left[\tilde{R}_j(X_i)\right]_{g(k)}\left(Y_i\mathbb{1}\{Y_i\leq t_n\} - \mathbb{E}\left[Y_i\mathbb{1}\{Y_i\leq t_n\}|X_i\right]\right), \\
T_{ij}(k) &= \mathbb{1}_{P_j}(X_i)\left[\tilde{R}_j(X_i)\right]_{g(k)}\left(Y_i\mathbb{1}\{Y_i > t_n\} - \mathbb{E}\left[Y_i\mathbb{1}\{Y_i > t_n\}|X_i\right]\right).
\end{aligned}$$

For the truncated term, since $|H_{ij}(k)| \leq t_n$ by construction and $\mathbb{E}\left[H_{ij}(k)^2\right] \leq Cq_j$, applying

Bernstein's inequality and $q_j \asymp J_n^{-d}$, we find that for fixed $k \in \mathbb{Z}_+^d$:

$$J_n^d \max_{1\leq j\leq J_n^d} \mathbb{P}\left[\left|\sum_{i=1}^n H_{ij}(k)\right| > nq_j r_n \varepsilon\right] \leq CJ_n^d \max_{1\leq j\leq J_n^d} \exp\left\{-C\frac{(nq_j r_n\varepsilon)^2}{nq_j + t_n nq_j r_n \varepsilon}\right\}$$
$$\leq C\exp\left\{\log(J_n^d)\left[1 - C\frac{nr_n^2(J_n^d\log(J_n^d))^{-1}\varepsilon^2}{1+t_n r_n\varepsilon}\right]\right\}.$$

By $\xi \in [0,1]$ and the rate restriction of the Theorem, the above probability can be made arbitrarily small for ε large enough, as:

$$\frac{n}{J_n^d\log(J_n^d)}r_n^2 = \frac{J_n^{d(1-\xi)}}{\log(J_n^d)^{1-\xi}} \geq 1, \quad \text{and,} \quad \frac{t_n}{r_n}\frac{J_n^d\log(J_n^d)}{n} = \left(\frac{J_n^{d\xi(1+2/\eta)}\log(J_n^d)^{2-\xi(1+2/\eta)}}{n}\right)^{1/2} = O(1).$$

For the tails, by Markov's inequality, $\mathbb{E}[T_{ij}(k)] = 0$, Lemma A.1, Assumption 1(c), and $q_j \asymp J_n^{-d}$:

$$J_n^d \max_{1\leq j\leq J_n^d} \mathbb{P}\left[\left|\sum_{i=1}^n T_{ij}(k)\right| > nq_j r_n\varepsilon\right] \leq CJ_n^d \max_{1\leq j\leq J_n^d}\frac{1}{(nq_j r_n\varepsilon)^2}\mathbb{E}\left[\left|\sum_{i=1}^n T_{ij}(k)\right|^2\right]$$
$$\leq C\frac{J_n^d}{nr_n^2\varepsilon^2}\max_{1\leq j\leq J_n^d}\frac{1}{q_j^2}\mathbb{E}\left[\mathbb{1}_{P_j}(X_i)\left|\left[\tilde{R}_j(X_i)\right]_{g(k)} Y_i \mathbb{1}\{Y_i > t_n\}\right|^2\right]$$
$$\leq C\frac{J_n^d}{nr_n^2 t_n^\eta\varepsilon^2}\max_{1\leq j\leq J_n^d}\frac{1}{q_j^2}\mathbb{E}\left[\mathbb{1}_{P_j}(X_i)\mathbb{E}\left[|Y_i|^{2+\eta}\,\middle|\, X_i\right]\right]$$
$$\leq C\frac{J_n^{2d}}{nr_n^2 t_n^\eta\varepsilon^2}.$$

This is arbitrarily small for large enough ε, since:

$$\frac{J_n^{2d}}{nr_n^2 t_n^\eta} = \frac{J_n^{2d}}{n}\frac{n}{J_n^{d(2-\xi)}\log(J_n^d)^\xi}\frac{\log(J_n^d)^\xi}{J_n^{d\xi}} = 1.$$

The two bounds do not depend on k, and hence by Boole's inequality and K constant,

$$\mathbb{P}\left[\max_{1\leq j\leq J_n^d}\left|\tilde{R}_j'(Y-G)/(nq_j)\right| > r_n\varepsilon\right] \leq CJ_n^d \max_{1\leq j\leq J_n^d}\max_{[k]\leq K-1}\mathbb{P}\left[\left|\sum_{i=1}^n H_{ij}(k)\right| > nq_j r_n\varepsilon\right]$$
$$+ CJ_n^d \max_{1\leq j\leq J_n^d}\max_{[k]\leq K-1}\mathbb{P}\left[\left|\sum_{i=1}^n T_{ij}(k)\right| > nq_j r_n\varepsilon\right],$$

which is arbitrarily small for ε large enough.

The conclusion will hold almost surely by the Borel-Cantelli Lemma if we find sequences $r_n \to 0$

and $t_n \to \infty$ such that

$$\frac{n}{J_n^d \log(J_n^d)} r_n^2 \not\to 0, \qquad \frac{t_n}{r_n} \frac{J_n^d \log(J_n^d)}{n} \not\to \infty, \quad \text{and,} \quad \sum_{n=1}^{\infty} \frac{J_n^{2d}}{n r_n^2 t_n^\eta} < \infty.$$

For r_n in the statement of the Lemma, the first requirement is satisfied as above. For $J_n^d \asymp (n/\log(n))^\gamma$ and $t_n = n^\tau$, $\tau > 0$, the second and third conditions above require $(1+\xi\gamma)/\eta < \tau < (1-\xi\gamma)/2$. This interval is nonempty since by assumption $\eta > 2\left(\frac{1+\xi\gamma}{1-\xi\gamma}\right)$. □

A.2 Convergence Rates

Proof of Theorem 1. Define $\mathbb{1}_{n,j} = \mathbb{1}\{\lambda_{\min}(\hat\Omega_j) \geq C\}$ for some positive constant C, where $\lambda_{\min}(\hat\Omega_j)$ is the smallest eigenvalue. In the proofs that follow we will redefine the notation $\hat\mu(x) = \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \tilde R_j(x)'\hat\beta_j$ (cf. Eqn. (1)). As $\min_{1\leq j\leq J_n^d} \mathbb{1}_{n,j} = 1$ w.p.a. 1 by Lemma A.4, this distinction vanishes asymptotically. For β_j^0 as in Lemma A.2 and $G = (\mu(X_1), \ldots, \mu(X_n))'$:

$$\begin{aligned}
\max_{m:[m]\leq s} \left\| \partial^m \hat\mu - \partial^m \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j}\mu_j \right\|_2^2 &= \max_{m:[m]\leq s} \left\| \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \left[(\partial^m \tilde R_j(\cdot))'\hat\Omega_j^{-1}\tilde R_j' Y/(nq_j) - \partial^m \mu_j(\cdot) \right] \right\|_2^2 \\
&\leq \max_{m:[m]\leq s} 3 \sum_{j=1}^{J_n^d} \left\| \mathbb{1}_{n,j} (\partial^m \tilde R_j(\cdot))'\hat\Omega_j^{-1}\tilde R_j'(Y-G)/(nq_j) \right\|_2^2 && (T_{n1}) \\
&\quad + \max_{m:[m]\leq s} 3 \sum_{j=1}^{J_n^d} \left\| \mathbb{1}_{n,j} (\partial^m \tilde R_j(\cdot))'\hat\Omega_j^{-1}\tilde R_j'(G - \tilde R_j\beta_j^0)/(nq_j) \right\|_2^2 && (T_{n2}) \\
&\quad + \max_{m:[m]\leq s} 3 \sum_{j=1}^{J_n^d} \left\| \mathbb{1}_{n,j} \left[(\partial^m \tilde R_j(\cdot))'\beta_j^0 - \partial^m \mu_j(\cdot) \right] \right\|_2^2 . && (T_{n3})
\end{aligned}$$

The proof proceeds by bounding T_{n1}–T_{n3}. To begin, observe that by properties of the trace operator, Assumption 1(c), $\tilde R_j(\tilde R_j'\tilde R_j)^{-1}\tilde R_j'$ idempotent, K fixed, and $q_j \asymp J_n^{-d}$,

$$\begin{aligned}
\mathbb{E}\left[\left| \mathbb{1}_{n,j}\hat\Omega_j^{-1/2}\tilde R_j'(Y-G)/(nq_j) \right|^2 \middle| \{X_i\} \right] &= \frac{\mathbb{1}_{n,j}}{nq_j} \operatorname{tr}\left\{ \mathbb{E}\left[(Y-G)'\tilde R_j \left(\tilde R_j'\tilde R_j\right)^{-1} \tilde R_j'(Y-G) \middle| \{X_i\} \right] \right\} \\
&= \frac{\mathbb{1}_{n,j}}{nq_j} \operatorname{tr}\left\{ \tilde R_j \left(\tilde R_j'\tilde R_j\right)^{-1} \tilde R_j' \mathbb{E}\left[(Y-G)(Y-G)' \middle| \{X_i\} \right] \right\} \\
&\leq C \frac{\mathbb{1}_{n,j}}{nq_j} \operatorname{tr}\left\{ \tilde R_j \left(\tilde R_j'\tilde R_j\right)^{-1} \tilde R_j' \right\} \\
&= C \frac{\mathbb{1}_{n,j}}{nq_j} \operatorname{tr}\left\{ \left(\tilde R_j'\tilde R_j\right)^{-1} \tilde R_j'\tilde R_j \right\}
\end{aligned}$$

$$\leq \frac{Cg^*}{nq_j} \leq \frac{CJ_n^d}{n}. \tag{A.2}$$

This bound is uniform in $1 \leq j \leq J_n^d$, and hence:

$$\mathbb{E}\left[\sum_{j=1}^{J_n^d} q_j \mathbb{1}_{n,j} \left|\frac{\hat{\Omega}_j^{-1/2}\tilde{R}_j'(Y-G)}{nq_j}\right|^2\right] \leq \max_{1\leq j\leq J_n^d} \mathbb{E}\left[\mathbb{1}_{n,j}\left|\frac{\hat{\Omega}_j^{-1/2}\tilde{R}_j'(Y-G)}{nq_j}\right|^2\right] \sum_{j=1}^{J_n^d} q_j = O\left(J_n^d/n\right).$$

So by Markov's inequality $\sum_{j=1}^{J_n^d} q_j \mathbb{1}_{n,j}\left|\hat{\Omega}_j^{-1/2}\tilde{R}_j'(Y-G)/(nq_j)\right|^2 = O_p\left(J_n^d/n\right)$. Using this result, that $q_j = \int_{P_j} f(x)dx$, Lemmas A.1 and A.4, and because the differentiation only affects the basis at the point of evaluation, we have the following bound:

$$T_{n1} \leq \left(\max_{1\leq j\leq J_n^d} \max_{m:[m]\leq s} \left\|\partial^m \tilde{R}_j(\cdot)\right\|_\infty^2\right)\left(\max_{1\leq j\leq J_n^d} \mathbb{1}_{n,j}\left|\hat{\Omega}_j^{-1}\right|\right)\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j}\left|\hat{\Omega}_j^{-1/2}\frac{\tilde{R}_j'(Y-G)}{nq_j}\right|^2 \int_{P_j} f(x)dx$$
$$= O\left(J_n^{2s}\right) O_p(1) O_p\left(J_n^d/n\right) = O_p\left(J_n^{d+2s}/n\right). \tag{A.3}$$

By Boole's and Bernstein's inequality and the condition of Theorem 1, the random variables $\mathbb{1}_{P_j}(X_i)$ satisfy the following, as $\mathbb{1}_{P_j}(X_i) \leq 1$ and $\mathbb{E}\left[\mathbb{1}_{P_j}(X)^2\right] = q_j$:

$$\mathbb{P}\left[\max_{1\leq j\leq J_n^d} \sum_{i=1}^n (\mathbb{1}_{P_j}(X_i) - q_j) > nq_j\varepsilon\right] \leq CJ_n^d \max_{1\leq j\leq J_n^d} \exp\left\{-C\frac{nq_j\varepsilon^2}{1+\varepsilon}\right\}$$
$$\leq C\exp\left\{\log(J_n^d)\left[1 - C\frac{n}{J_n^d \log(J_n^d)}\frac{\varepsilon^2}{1+\varepsilon}\right]\right\} \to 0. \tag{A.4}$$

Therefore, by $\tilde{R}_j(\tilde{R}_j'\tilde{R}_j)^{-1}\tilde{R}_j'$ idempotent and Lemma A.2:

$$\begin{aligned}
&\max_{1\leq j\leq J_n^d}\left|\mathbb{1}_{n,j}\hat{\Omega}_j^{-1/2}\tilde{R}_j'\left(G - \tilde{R}_j\beta_j^0\right)/(nq_j)\right|^2 \\
&\quad= \max_{1\leq j\leq J_n^d} \mathbb{1}_{n,j}\left|\left(G-\tilde{R}_j\beta_j^0\right)'\tilde{R}_j(\tilde{R}_j'\tilde{R}_j)^{-1}\tilde{R}_j'\left(G-\tilde{R}_j\beta_j^0\right)/(nq_j)\right| \\
&\quad\leq \max_{1\leq j\leq J_n^d}\left|\left(G-\tilde{R}_j\beta_j^0\right)'\left(G-\tilde{R}_j\beta_j^0\right)/(nq_j)\right| \\
&\quad= \max_{1\leq j\leq J_n^d}\frac{1}{nq_j}\sum_{i=1}^n \mathbb{1}_{P_j}(X_i)\left(\mu(X_i) - \tilde{R}_j(X_i)'\beta_j^0\right)^2 \\
&\quad\leq \max_{1\leq j\leq J_n^d}\left\|\mathbb{1}_{P_j}(\cdot)\left(\mu(\cdot) - \tilde{R}_j(\cdot)'\beta_j^0\right)\right\|_\infty^2 \max_{1\leq j\leq J_n^d}\frac{1}{nq_j}\sum_{i=1}^n \mathbb{1}_{P_j}(X_i) = O_p\left(J_n^{-2((S+\alpha)\wedge K)}\right).
\end{aligned} \tag{A.5}$$

The first inequality follows by the fact that for a vector v and idempotent matrix P, $v'Pv =$

$v'v - v'(I-P)v = v'v - |(I-P)v| \le v'v$, since norms are nonnegative. And so for T_{n2}, by the above result, Lemmas A.1 and A.4, and $\sum_{j=1}^{J_n^d} \int_{P_j} f(x)dx = \int_{\mathcal{X}} f(x)dx = 1$, we have

$$\begin{aligned} T_{n2} &\le \left(\max_{1\le j\le J_n^d} \max_{m:[m]\le s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_\infty^2 \right) \left(\max_{1\le j\le J_n^d} \mathbb{1}_{n,j} \left| \hat{\Omega}_j^{-1} \right| \right) \\ &\quad \times \left(\max_{1\le j\le J_n^d} \left| \mathbb{1}_{n,j} \hat{\Omega}_j^{-1/2} \tilde{R}_j' \left(G - \tilde{R}_j \beta_j^0 \right) / (nq_j) \right|^2 \right) \sum_{j=1}^{J_n^d} \int_{P_j} f(x)dx \\ &\le O\left(J_n^{2s} \right) O_p(1) O_p\left(J_n^{-2((S+\alpha)\wedge K)} \right) = O_p\left(J_n^{-2((S+\alpha)\wedge K - s)} \right). \end{aligned} \tag{A.6}$$

Finally, Lemma A.2 immediately gives:

$$T_{n3} \le \max_{1\le j\le J_n^d} \max_{m:[m]\le s} \left\| \mathbb{1}_{n,j} \left((\partial^m \tilde{R}_j(\cdot))' \beta_j^0 - \partial^m \mu_j(\cdot) \right) \right\|_\infty^2 \sum_{j=1}^{J_n^d} \int_{P_j} f(x)dx = O\left(J_n^{-2((S+\alpha)\wedge K - s)} \right), \tag{A.7}$$

again using that $\sum_{j=1}^{J_n^d} \int_{P_j} f(x)dx = 1$.

Combining the bounds (A.3), (A.6), and (A.7), the result follows from $\min_{1\le j\le J_n^d} \mathbb{1}_{n,j} = 1$ w.p.a. 1 by Lemma A.4. □

Proof of Theorem 2. For β_j^0 as in Lemma A.2:

$$\begin{aligned} \max_{m:[m]\le s} \left\| \partial^m \hat{\mu} - \partial^m \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \mu_j \right\|_\infty^2 &= \max_{1\le j\le J_n^d} \max_{m:[m]\le s} \left\| \mathbb{1}_{n,j} \left((\partial^m \tilde{R}_j(\cdot))' (\tilde{R}_j' \tilde{R}_j)^{-1} \tilde{R}_j' Y - \partial^m \mu_j(\cdot) \right) \right\|_\infty^2 \\ &\le \max_{1\le j\le J_n^d} \max_{m:[m]\le s} 3 \left\| \mathbb{1}_{n,j} \left((\partial^m \tilde{R}_j(\cdot))' \hat{\Omega}_j^{-1} \tilde{R}_j' (Y-G)/(nq_j) \right) \right\|_\infty^2 \\ &\quad + \max_{1\le j\le J_n^d} \max_{m:[m]\le s} 3 \left\| \mathbb{1}_{n,j} \left((\partial^m \tilde{R}_j(\cdot))' \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0)/(nq_j) \right) \right\|_\infty^2 \\ &\quad + \max_{1\le j\le J_n^d} \max_{m:[m]\le s} 3 \left\| \mathbb{1}_{n,j} \left((\partial^m \tilde{R}_j(\cdot))' \beta_j^0 - \partial^m \mu_j(\cdot) \right) \right\|_\infty^2. \end{aligned}$$

Using Lemmas A.1, A.4, and A.5, the first term is bounded by:

$$\begin{aligned} &C \left(\max_{1\le j\le J_n^d} \max_{m:[m]\le s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_\infty^2 \right) \left(\max_{1\le j\le J_n^d} \left| \mathbb{1}_{n,j} \hat{\Omega}_j^{-1} \right| \right) \left\| \tilde{R}_j' (Y-G)/(nq_j) \right\|_\infty^2 \\ &\qquad = O\left(J_n^{2s} \right) O_p(1) O_p\left(\frac{J_n^{d(2-\xi)} \log(J_n^d)^\xi}{n} \right). \end{aligned}$$

Using Lemmas A.1 and A.4 and Eqn. (A.5), the second term is bounded by:

$$\left(\max_{1\leq j\leq J_n^d}\max_{m:[m]\leq s}\left\|\partial^m\tilde{R}_j(\cdot)\right\|_\infty^2\right)\left(\max_{1\leq j\leq J_n^d}\mathbb{1}_{n,j}\left|\hat{\Omega}_j^{-1}\right|\right)\left(\max_{1\leq j\leq J_n^d}\left|\mathbb{1}_{n,j}\hat{\Omega}_j^{-1/2}\tilde{R}_j'\left(G-\tilde{R}_j\beta_j^0\right)/(nq_j)\right|^2\right)$$
$$= O\left(J_n^{2s}\right)O_p(1)O_p\left(J_n^{-2((S+\alpha)\wedge K)}\right).$$

Finally, the rate for the third term is given in Lemma A.2, since $\mathbb{1}_{n,j}\leq 1$. Adding these three rates completes the proof, as $\min_{1\leq j\leq J_n^d}\mathbb{1}_{n,j}=1$ w.p.a. 1 by Lemma A.4. □

We now demonstrate a version of Theorem 2 that holds with probability one.

Theorem A.1. *Suppose the conditions of Theorem 1 hold. If, in addition, for some $\xi\in[0,1\wedge\eta]$ the partition satisfies $J_n^d\asymp(n/\log(n))^\gamma$, $\gamma\in(0,1)$ and $\eta>2(1+\xi\gamma)/(1-\xi\gamma)$, then for $s\leq S\wedge(K-1)$:*

$$\max_{m:[m]\leq s}\|\partial^m\hat{\mu}-\partial^m\mu\|_\infty^2=O_{as}\left(\frac{J_n^{(2-\xi)d+2s}\log(J_n^d)^\xi}{n}+J_n^{-2((S+\alpha)\wedge K-s)}\right).$$

Proof of Theorem A.1. First observe that the rate restriction on J_n given implies that of Theorem 2. This holds because by the assumption on η,

$$\eta>2\frac{1+\gamma\xi}{1-\gamma\xi}>2\frac{\gamma\xi}{1-\gamma\xi},$$

and hence $\gamma\xi(1+2/\eta)\leq 1$. The exponential bound of (A.4) and $n^{-1}J_n^d\log(J_n^d)\to 0$ gives

$$\max_{1\leq j\leq J_n^d}\frac{1}{nq_j}\sum_{i=1}^n\mathbb{1}_{P_j}(X_i)=O_{as}(1).$$

Hence Eqn. (A.5) and the steps of Eqn. (A.6) hold almost surely. Coupled with the second conclusion in Lemma A.5, the proof of Theorem 2 can be strengthened to hold with probability one, as $\min_{1\leq j\leq J_n^d}\mathbb{1}_{n,j}=1$ w.p.a. 1 using the almost sure Lemma A.4. □

A.3 Asymptotic Mean-Square Error

We first give three lemmas necessary for results.

Lemma A.6. *Let the conditions of Theorem 1 hold and $g(\cdot)$ be continuous on $\mathcal{X}$. Then for $h_j(x)=\mathbb{1}_{P_j}(x)h(x)$, with remainder uniform in $1\leq j\leq J_n^d$:*

$$\int_{P_j}h(z)g(z)dz=g(\overline{p}_j)\int_{P_j}h(z)dz+\max_{1\leq j\leq J_n^d}\|h_j(\cdot)\|_\infty(o(J_n^{-d})).$$

Further, if $g(\cdot)$ is Lipschitz continuous, then

$$\int_{P_j} h(z)g(z)dz = g(\overline{p}_j)\int_{P_j} h(z)dz + \max_{1\leq j\leq J_n^d}\|h_j(\cdot)\|_\infty(O(J_n^{-d-1})).$$

Proof. First, write

$$\int_{P_j} h(z)g(z)dz = g(\overline{p}_j)\int_{P_j} h(z)dz + \int_{P_j} h(z)[g(z)-g(\overline{p}_j)]dz.$$

By continuity, the remainder is bounded by:

$$\begin{aligned}\max_{1\leq j\leq J_n^d}\int_{P_j}|h(z)|\left|g(z)-g(\overline{p}_j)\right|dz &\leq \max_{1\leq j\leq J_n^d}\sup_{x\in P_j}\left|g(\overline{p}_j)-g(x)\right|\max_{1\leq j\leq J_n^d}\|h_j(\cdot)\|_\infty\int_{P_j}dz\\ &= o(1)\max_{1\leq j\leq J_n^d}\|h_j(\cdot)\|_\infty\max_{1\leq j\leq J_n^d}\mathrm{vol}\,(P_j) = \max_{1\leq j\leq J_n^d}\|h_j(\cdot)\|_\infty o\left(J_n^{-d}\right).\end{aligned}$$

The second conclusion follows from the same steps, but the rate is obtained from the Lipschitz continuity because $\max_{1\leq j\leq J_n^d}\sup_{x\in P_j}\left|g(\overline{p}_j)-g(x)\right| \leq C|p_j^*-\overline{p}_j| = O(J_n^{-1})$. □

Lemma A.7. *Let the conditions of Theorem 1 hold. If $g(\cdot)$ is continuous on $\mathcal{X}$, then:*

$$\sum_{j=1}^{J_n^d} g(\overline{p}_j)\,\mathrm{vol}\,(P_j) = \int_{\mathcal{X}} g(z)dz + o(1).$$

Further, if $g(\cdot)$ is Lipschitz continuous, then the remainder is $O(J_n^{-1})$.

Proof. This is a standard Riemann sum argument: the result follows as $g(\cdot)$ is continuous and $\mathcal{X}$ is compact.

$$\begin{aligned}\sum_{j=1}^{J_n^d} g(\overline{p}_j)\,\mathrm{vol}\,(P_j) - \int_{\mathcal{X}} g(z)dz &= \sum_{j=1}^{J_n^d}\left(g(\overline{p}_j)\,\mathrm{vol}\,(P_j) - \int_{P_j} g(z)dz\right)\\ &= \sum_{j=1}^{J_n^d}\left(g(\overline{p}_j)\int_{P_j}dz - \int_{P_j} g(z)dz\right)\\ &= \sum_{j=1}^{J_n^d}\int_{P_j}(g(\overline{p}_j)-g(z))dz.\end{aligned}$$

Then, as $\sum_{j=1}^{J_n^d}\int_{P_j}dz = \int_{\mathcal{X}}dz = \mathrm{vol}\,(\mathcal{X})$, and $\mathcal{X}$ is compact, we have by continuity that the magnitude of this remainder is bounded by

$$\sum_{j=1}^{J_n^d} \int_{P_j} \big|g(\overline{p}_j) - g(z)\big| \, dz \leq \max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \big|g(\overline{p}_j) - g(x)\big| \sum_{j=1}^{J_n^d} \int_{P_j} dz$$

$$= o(1) \operatorname{vol}(\mathcal{X}) = o(1).$$

The second conclusion follows from the same steps, but the rate is obtained from the Lipschitz continuity because $\max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \big|g(\overline{p}_j) - g(x)\big| \leq C|p_j^* - \overline{p}_j| = O(J_n^{-1})$. □

Lemma A.8. *Under the conditions of Theorem 3, for Γ_j defined Eqn.* (4) *and any $k \in \mathbb{Z}_+^d$:*

$$(a) \qquad \max_{1 \leq j \leq J_n^d} \left| \frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \sigma^2(X_i) - \Gamma_j \right|^2 = O_p\left(\frac{J_n^d \log(J_n^d)}{n} \right);$$

$$(b) \qquad \max_{1 \leq j \leq J_n^d} \left| \frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \frac{(X_i - \overline{p}_j)^k}{(p_j^* - \overline{p}_j)^k} - \frac{1}{q_j} \mathbb{E}\left[\tilde{R}_j(X) \frac{(X_i - \overline{p}_j)^k}{(p_j^* - \overline{p}_j)^k} \right] \right|^2 = O_p\left(\frac{J_n^d \log(J_n^d)}{n} \right).$$

Proof. Both results follow identically to Lemma A.4, the conditions of which are met as $\sigma^2(x)$ is bounded by Assumption 2(a) and $|(X_i - \overline{p}_j)^k / (p_j^* - \overline{p}_j)^k| \leq 1$ by construction. The proof goes through essentially as written, with the appropriate notational changes to $W_{ij}(k, \tilde{k})$. □

Proof of Theorem 3. We first give some notation and facts used repeatedly throughout. With a slight abuse notation, let $|\mathcal{X}|^k = \prod_{\ell=1}^d |\mathcal{X}_\ell|^{k_\ell}$; this is distinct from $\operatorname{vol}(\mathcal{X})$ unless $k_\ell = 1$ for all $\ell = 1, \ldots, d$. Let $\mathcal{U} = \times_{\ell=1}^d [-1, 1]$ be the Cartesian product of d copies of the interval $[-1, 1]$. Under the conditions placed on the partition and Assumption 1(b), $\operatorname{vol}(P_j) = \operatorname{vol}(\mathcal{X}) / J_n^d$. We frequently use the change of variables $z_\ell = (x_\ell - \overline{p}_{\ell,j})/(p_{\ell,j} - \overline{p}_{\ell,j})$, $\ell = 1, \ldots, d$, the Jacobian of which is $\prod_{\ell=1}^d (p_{\ell,j} - \overline{p}_{\ell,j}) = 2^{-d} \operatorname{vol}(P_j)$. Recall from Lemma A.2 that entry $g(k)$ of $\partial^m \tilde{R}_j(x)$ is given by:

$$\mathbb{1}\{m \leq k\} \frac{k!}{(k-m)!} \frac{(x - \overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k},$$

whereas the same entry of $\partial R_j(x) = (k!/(k-m)!) x^{k-m}$. Finally, because the partition is evenly spaced, for any $k \in \mathbb{Z}_+^d$:

$$(p_j^* - \overline{p}_j)^k = \prod_{\ell=1}^d \frac{(p_{\ell,j} - p_{\ell,j-1})^{k_\ell}}{2^{k_\ell}} = \prod_{\ell=1}^d (|\mathcal{X}_\ell|/(2J_n))^{k_\ell} = (2J_n)^{-[k]} |\mathcal{X}|^k.$$

Using Lemma A.6 and the change of variables above, we get the following results which are used below: first,

$$\int_{\mathcal{X}}(\partial^m \tilde{R}_j(x))(x-\overline{p}_j)^{k-m} w(x)dx = w(\overline{p}_j)\int_{P_j}(\partial^m \tilde{R}_j(x))(x-\overline{p}_j)^{k-m}dx + o(J_n^{-d-K})$$
$$= 2^{-d} w(\overline{p}_j)(p_j^* - \overline{p}_j)^{k-2m}\operatorname{vol}(P_j)\int_{\mathcal{U}}(\partial^m R(z))z^k dz + o(J_n^{-d-K}),$$

which also holds for $w(x)=f(x)$ or $m=0$; second,

$$\Omega_j = \frac{2^{-d}}{q_j} f(\overline{p}_j)\operatorname{vol}(P_j)\int_{\mathcal{U}} R(z)R(z)'dz + o(J_n^{-d});$$

and finally,

$$\int_X \left(\partial^m \tilde{R}_j(x)\right)\left(\partial^m \tilde{R}_j(x)\right)' w(x)dx = \frac{2^{-d} w(\overline{p}_j)\operatorname{vol}(P_j)}{(p_j^* - \overline{p}_j)^{2m}}\int_{\mathcal{U}}(\partial^m R(z))\,(\partial^m R(z))'\,dz + o(J_n^{-d-2[m]}),$$

where we have applied Lemma A.1 to bound $\partial^m \tilde{R}_j(x)$.

Recall that $\mathbf{X}_n = (X_1,\ldots,X_n)'$ and expand as follows:

$$\int_X \mathbb{E}\left[(\partial^m \hat{\mu}(x) - \partial^m \mu(x))^2\,|\mathbf{X}_n\right] w(x)dx = \int_{\mathcal{X}}\left\{\mathbb{V}\left[\partial^m \hat{\mu}(x)\mid \mathbf{X}_n\right] + \left(\mathbb{E}\left[\partial^m \hat{\mu}(x)|\mathbf{X}_n\right] - \partial^m \mu(x)\right)^2\right\} w(x)dx.$$

First consider the variance term. By Lemma A.6, $\Gamma_j = \sigma^2(\overline{p}_j)\Omega_j + o(J_n^{-d})$. Applying this result and Lemmas A.1, A.4, and A.8(a), we have:

$$\begin{aligned}
&\mathbb{V}\left[\sum_{j=1}^{J_n^d}\left(\partial^m \tilde{R}_j(x)\right)' \mathbb{1}_{n,j}\hat{\Omega}_j^{-1}\tilde{R}_j Y/(nq_j)|\mathbf{X}_n\right]\\
&= \sum_{j=1}^{J_n^d}\frac{1}{nq_j}\left(\partial^m \tilde{R}_j(x)\right)' \mathbb{1}_{n,j}\hat{\Omega}_j^{-1}\left(\frac{1}{nq_j}\sum_{i=1}^n \tilde{R}_j(X_i)\tilde{R}_j(X_i)'\sigma^2(X_i)\right)\hat{\Omega}_j^{-1}\left(\partial^m \tilde{R}_j(x)'\right)\\
&= \sum_{j=1}^{J_n^d}\frac{1}{nq_j}\left(\partial^m \tilde{R}_j(x)\right)'\Omega_j^{-1}\Gamma_j\Omega_j^{-1}\left(\partial^m \tilde{R}_j(x)\right) + O_p\left(\frac{J_n^{d+2[m]}}{n}\frac{J_n^d\log(J_n^d)}{n}\right)\\
&= \sum_{j=1}^{J_n^d}\frac{1}{nq_j}\sigma^2(\overline{p}_j)\left(\partial^m \tilde{R}_j(x)\right)'\Omega_j^{-1}\Omega_j\Omega_j^{-1}\left(\partial^m \tilde{R}_j(x)\right) + o_p\left(\frac{J_n^{d+2[m]}}{n}\right)\\
&= \sum_{j=1}^{J_n^d}\frac{1}{nq_j}\sigma^2(\overline{p}_j)\left(\partial^m \tilde{R}_j(x)\right)'\Omega_j^{-1}\left(\partial^m \tilde{R}_j(x)\right) + o_p\left(\frac{J_n^{d+2[m]}}{n}\right)
\end{aligned}$$

$$= \sum_{j=1}^{J_n^d} \frac{1}{nq_j}\sigma^2(\overline{p}_j)\,\mathrm{tr}\left\{\left(\partial^m \tilde{R}_j(x)\right)'\Omega_j^{-1}\left(\partial^m \tilde{R}_j(x)\right)\right\} + o_p\left(\frac{J_n^{d+2[m]}}{n}\right)$$
$$= \sum_{j=1}^{J_n^d} \frac{1}{nq_j}\sigma^2(\overline{p}_j)\,\mathrm{tr}\left\{\Omega_j^{-1}\left(\partial^m \tilde{R}_j(x)\right)\left(\partial^m \tilde{R}_j(x)\right)'\right\} + o_p\left(\frac{J_n^{d+2[m]}}{n}\right).$$

Integrating the above expression, applying Lemma A.6, the above facts and change of variables, and Lemma A.7 (under Assumption 2(a)), we have:

$$\sum_{j=1}^{J_n^d} \frac{1}{nq_j}\sigma^2(\overline{p}_j)\,\mathrm{tr}\left\{\Omega_j^{-1}\int_X \left(\partial^m \tilde{R}_j(x)\right)\left(\partial^m \tilde{R}_j(x)\right)' w(x)dx\right\} + o_p\left(\frac{J_n^{d+2[m]}}{n}\right)$$
$$= \frac{1}{n}\sum_{j=1}^{J_n^d} \frac{w(\overline{p}_j)}{f(\overline{p}_j)}\sigma^2(\overline{p}_j)\frac{1}{(p_j^* - \overline{p}_j)^{2m}}\,\mathrm{tr}\left\{\left(\int_{\mathcal{U}} R(z)R(z)'dz\right)^{-1}\int_{\mathcal{U}}\left(\partial^m R(z)\right)\left(\partial^m R(z)\right)'dz\right\}$$
$$+ o_p\left(\frac{J_n^{d+2[m]}}{n}\right)$$
$$= \frac{1}{n}\frac{J_n^d}{\mathrm{vol}\,(\mathcal{X})}\frac{(2J_n)^{2[m]}}{|\mathcal{X}|^{2m}}\sum_{j=1}^{J_n^d} \frac{w(\overline{p}_j)}{f(\overline{p}_j)}\sigma^2(\overline{p}_j)\,\mathrm{vol}\,(P_j)\,\mathrm{tr}\left\{\left(\int_{\mathcal{U}} R(z)R(z)'dz\right)^{-1}\int_{\mathcal{U}}\left(\partial^m R(z)\right)\left(\partial^m R(z)\right)'dz\right\}$$
$$+ o_p\left(\frac{J_n^{d+2[m]}}{n}\right)$$
$$= \frac{J_n^{d+2[m]}}{n}\frac{2^{2[m]}}{|\mathcal{X}|^{2m}\,\mathrm{vol}\,(\mathcal{X})}\left(\int_{\mathcal{X}}\frac{\sigma^2(x)}{f(x)}w(x)dx\right)\mathrm{tr}\left\{\left(\int_{\mathcal{U}} R(z)R(z)'dz\right)^{-1}\int_{\mathcal{U}}\left(\partial^m R(z)\right)\left(\partial^m R(z)\right)'dz\right\}$$
$$\times [1 + o(1)] + o_p\left(\frac{J_n^{d+2[m]}}{n}\right).$$

Next, recall that $K = S + 1$ and that $\tilde{R}_j(x)$ is of degree $K - 1$. From Lemma A.2, under Assumption 2(b), we have that $\partial^m \mu_j(x)$ satisfies the Taylor expansion for $x \in P_j$:

$$\partial^m \mu_j(x) - \partial^m \tilde{R}_j(x)'\beta_j^0 = \sum_{k:[k]=K}\left(\partial^k \mu_j(\overline{p}_j)\right)\frac{(x - \overline{p}_j)^{k-m}}{(k-m)!} + o(J_n^{-(K-[m])}) \equiv T_{K,j,m}(x) + o(J_n^{-(K-[m])}),$$

where β_j^0 does not depend on m and the remainder is uniform over $1 \le j \le J_n^d$. The final equality defines $T_{K,j,m}(x)$ as the leading term. Therefore, by Lemmas A.4 and A.8,

$$\sum_{j=1}^{J_n^d}\partial^m \tilde{R}_j(x)'\mathbb{1}_{n,j}(\tilde{R}_j'\tilde{R}_j)^{-1}\sum_{i=1}^{n}\tilde{R}_j(X_i)\mu(X_i) - \sum_{j=1}^{J_n^d}\partial^m \mu_j(x)$$

$$
\begin{aligned}
&= \sum_{j=1}^{J_n^d} \left(\partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \beta_j^0 - \partial^m \mu_j(x) \right) \\
&\qquad + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left(T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \\
&= \sum_{j=1}^{J_n^d} \left(\partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \left(\sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \right) \beta_j^0 - \partial^m \mu_j(x) \right) \\
&\qquad + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left(T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \\
&= \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \left(\partial^m \tilde{R}_j(x)' \beta_j^0 - \partial^m \mu_j(x) \right) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) T_{K,j,0}(X_i) + o\left(J_n^{-(K-[m])} \right) \\
&= -\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \mathbb{1}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) T_{K,j,0}(X_i) \\
&\qquad\qquad + o(J_n^{-(K-[m])}) \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \mathbb{1}_{P_j}(x) + o\left(J_n^{-(K-[m])} \right) \\
&= -\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \mathbb{1}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} \hat{\Omega}_j^{-1} \sum_{k:[k]=K} \frac{\partial^k \mu_j(\overline{p}_j)}{k!} \frac{1}{n q_j} \sum_{i=1}^n \tilde{R}_j(X_i)(X_i - \overline{p}_j)^k \\
&\qquad\qquad + o\left(J_n^{-(K-[m])} \right) \\
&= -\sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \frac{1}{q_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \sum_{k:[k]=K} \frac{\partial^k \mu_j(\overline{p}_j)}{k!} \mathbb{E}\left[\tilde{R}_j(X)(X - \overline{p}_j)^k \right] \\
&\qquad\qquad + O_p\left(J_n^{-(K-[m])} \frac{J_n^d \log(J_n^d)}{n} \right) + o\left(J_n^{-(K-[m])} \right) \\
&= \sum_{k:[k]=K} \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \left(\partial^k \mu_j(\overline{p}_j) \right) \left(-\frac{1}{(k-m)!} (x - \overline{p}_j)^{k-m} + \frac{1}{k!} \frac{1}{q_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X - \overline{p}_j)^k \right] \right) \\
&\qquad\qquad + o_p\left(J_n^{-(K-[m])} \right).
\end{aligned}
$$

Then since $\min_{1 \le j \le J_n^d} \mathbb{1}_{n,j} = 1$ w.p.a. 1 by Lemma A.4, the integrated, squared bias becomes:

$$
\int_{\mathcal{X}} \left(\mathbb{E}\left[\hat{\mu}(x) | \mathbf{X}_n \right] - \mu(x) \right)^2 w(x) dx
$$

$$
\begin{aligned}
&= \sum_{j=1}^{J_n^d} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \left(\partial^k \mu_j(\overline{p}_j)\right) \left(\partial^{\tilde{k}} \mu_j(\overline{p}_j)\right) \Bigg\{ \frac{1}{(k-m)!(\tilde{k}-m)!} \int_{P_j} (x-\overline{p}_j)^{k+\tilde{k}-2m} w(x) dx \\
&\quad + \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \int_{P_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X-\overline{p}_j)^k\right] \mathbb{E}\left[(X-\overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)'\right] \Omega_j^{-1} \partial^m \tilde{R}_j(x) w(x) dx \\
&\quad - \frac{1}{k!(\tilde{k}-m)!} \frac{1}{q_j} \int_{P_j} (x-\overline{p}_j)^{\tilde{k}-m} \partial^m \tilde{R}_j(x)' w(x) dx \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X-\overline{p}_j)^k\right] \\
&\quad - \frac{1}{\tilde{k}!(k-m)!} \frac{1}{q_j} \int_{P_j} (x-\overline{p}_j)^{k-m} \partial^m \tilde{R}_j(x)' w(x) dx \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X-\overline{p}_j)^{\tilde{k}}\right] \Bigg\} + o_p\left(J_n^{-2(K-[m])}\right) \\
&= \sum_{j=1}^{J_n^d} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \left(\partial^k \mu_j(\overline{p}_j)\right) \left(\partial^{\tilde{k}} \mu_j(\overline{p}_j)\right) \{B_1 + B_2 - B_3 - B_4\} + o_p\left(J_n^{-2(K-[m])}\right),
\end{aligned}
$$

where the final equality defines the terms B_1–B_4. We examine each in order, applying Lemma A.6 and the change of variables above. For the first term,

$$
\begin{aligned}
B_1 &= \frac{w(\overline{p}_j)}{(k-m)!(\tilde{k}-m)!} \int_{P_j} (x-\overline{p}_j)^{k+\tilde{k}-2m} dx + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right) \\
&= \frac{w(\overline{p}_j)(p_j^*-\overline{p}_j)^{k+\tilde{k}-2m}}{(k-m)!(\tilde{k}-m)!} \int_{P_j} \frac{(x-\overline{p}_j)^{k+\tilde{k}-2m}}{(p_j^*-\overline{p}_j)^{k+\tilde{k}-2m}} dx + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right) \\
&= \frac{(p_j^*-\overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j)}{2^d (k-m)!(\tilde{k}-m)!} \int_{\mathcal{U}} z^{k+\tilde{k}-2m} dz + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right).
\end{aligned}
$$

For the second, applying the results given above,

$$
\begin{aligned}
B_2 &= \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \int_{P_j} \operatorname{tr}\left\{ (\partial^m \tilde{R}_j(x))' \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X-\overline{p}_j)^k\right] \mathbb{E}\left[(X-\overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)'\right] \Omega_j^{-1} (\partial^m \tilde{R}_j(x)) \right\} w(x) dx \\
&= \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \operatorname{tr}\left\{ \Omega_j^{-1} \mathbb{E}\left[\tilde{R}_j(X)(X-\overline{p}_j)^k\right] \mathbb{E}\left[(X-\overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)'\right] \Omega_j^{-1} \int_{P_j} (\partial^m \tilde{R}_j(x))(\partial^m \tilde{R}_j(x))' w(x) dx \right\} \\
&= \frac{(p_j^*-\overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j)}{2^d k!\tilde{k}!} \operatorname{tr}\Bigg\{ \left(\int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} R(z) z^k dz \\
&\qquad \times \int_{\mathcal{U}} R(z)' z^{\tilde{k}} dz \left(\int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} \left(\partial^m R(z)\right) \left(\partial^m R(z)\right)' dz \Bigg\} + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right).
\end{aligned}
$$

Similarly,

$$
B_3 = \frac{(p_j^*-\overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j)}{2^d k!(\tilde{k}-m)!} \int_{\mathcal{U}} (\partial^m R(z))' z^{\tilde{k}-m} dz \left(\int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1}
$$

$$\times \int_{\mathcal{U}} R(z) z^k dz + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right).$$

Identical steps apply to B_4, with k and $\tilde{k}$ reversed.

All four terms have the common factor $(p_j^* - \overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j)$, which contains all dependence on the partition. By Lemma A.7, the facts at the outset, and that $[k] = [\tilde{k}] = K$,

$$\sum_{j=1}^{J_n^d} \left(\partial^k \mu_j(\overline{p}_j)\right)\left(\partial^{\tilde{k}} \mu_j(\overline{p}_j)\right)(p_j^* - \overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j)$$

$$= J_n^{-2(K-[m])} \frac{|\mathcal{X}|^{k+\tilde{k}-2m}}{2^{2(K-[m])}} \int_{\mathcal{X}} \left(\partial^k \mu_j(x)\right)\left(\partial^{\tilde{k}} \mu_j(x)\right) w(x) dx [1 + o(1)].$$

Combining all the above steps, if we define the two constants

$$\begin{aligned} \mathscr{V}_{K,d,m} = \frac{2^{2[m]}}{\operatorname{vol}(\mathcal{X})} \left(\prod_{\ell=1}^{d} |\mathcal{X}_\ell|^{-2m_\ell}\right)\left(\int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x) dx\right) \\ \times \operatorname{tr}\left\{\left(\int_{\mathcal{U}} R(z)R(z)' dz\right)^{-1} \int_{\mathcal{U}} \left(\partial^m R(z)\right)\left(\partial^m R(z)\right)' dz\right\} \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} \mathscr{B}_{K,d,m} = 2^{-2(K+d-[m])} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \left(\prod_{\ell=1}^{d} |\mathcal{X}_\ell|^{k_\ell+\tilde{k}_\ell-2m_\ell}\right)\left(\int_{\mathcal{X}} \left(\partial^k \mu(x)\right)\left(\partial^{\tilde{k}} \mu(x)\right) w(x) dx\right) \\ \times \Bigg\{ \frac{1}{(k-m)!(\tilde{k}-m)!} \int_{\mathcal{U}} z^{k+\tilde{k}-2m} dz \\ + \frac{1}{k!\tilde{k}!} \operatorname{tr}\Bigg[\left(\int_{\mathcal{U}} R(x)R(x)' dz\right)^{-1} \int_{\mathcal{U}} R(z) z^k dz \\ \times \int_{\mathcal{U}} R(z)' z^{\tilde{k}} dz \left(\int_{\mathcal{U}} R(z)R(z)' dz\right)^{-1} \int_{\mathcal{U}} \left(\partial^m R(z)\right)\left(\partial^m R(z)\right)' dz\Bigg] \\ - \frac{1}{k!(\tilde{k}-m)!} \int_{\mathcal{U}} (\partial^m R(z))' z^{\tilde{k}-m} dz \left(\int_{\mathcal{U}} R(z)R(z)' dz\right)^{-1} \int_{\mathcal{U}} R(z) z^k dz \\ - \frac{1}{\tilde{k}!(k-m)!} \int_{\mathcal{U}} (\partial^m R(z))' z^{k-m} dz \left(\int_{\mathcal{U}} R(z)R(z)' dz\right)^{-1} \int_{\mathcal{U}} R(z) z^{\tilde{k}} dz \Bigg\}, \end{aligned} \tag{A.9}$$

we obtain the final result, applying $\min_{1 \le j \le J_n^d} \mathbb{1}_{n,j} = 1$ w.p.a. 1 by Lemma A.4.

Finally, we give demonstrate simplifications of the above constants in specials cases. First, let

$[m] = 0$. In this case,

$$\operatorname{tr}\left\{ \left(\int_{\mathcal{U}} R_j(z) R_j(z)' dz \right)^{-1} \int_{\mathcal{U}} \left(\partial^m R_j(z)\right) \left(\partial^m R_j(z)\right)' dz \right\} = g^* = \dim(R(\cdot)).$$

Therefore

$$\mathscr{V}_{K,d,0} = \frac{\dim(R(\cdot))}{\operatorname{vol}(\mathcal{X})} \int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x) dx,$$

and

$$\begin{aligned} \mathscr{B}_{K,d,0} = \frac{1}{2^{2K+d}} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \frac{1}{k!\tilde{k}!} \left(\prod_{\ell=1}^{d} |\mathcal{X}_\ell|^{k_\ell + \tilde{k}_\ell} \right) \left\{ \int_{\mathcal{X}} \left(\partial^k \mu(x)\right) \left(\partial^{\tilde{k}} \mu(x)\right) w(x) dx \right\} \\ \times \left\{ \int_{\mathcal{U}} z^{k+\tilde{k}} dz - \int_{\mathcal{U}} R(z)' z^{\tilde{k}} dz \left(\int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} R(z) z^k dz \right\}. \end{aligned} \tag{A.10}$$

The variance constant is already considerably simplified when estimating the level of $\mu(x)$. For the bias, we examine two further specializations. First, if $m = 0$ and $d = 1$, then $k = \tilde{k} = K$, and so $\int_{-1}^{1} z^{2K} = \frac{2}{1+2K}$ and $\prod_{\ell=1}^{d} |\mathcal{X}_\ell|^{k_\ell + \tilde{k}_\ell} = \operatorname{vol}(\mathcal{X})^{2K}$. Hence

$$\begin{aligned} \mathscr{B}_{K,1,0} = \frac{\operatorname{vol}(\mathcal{X})^{2K}}{2^{2K+1}(K!)^2} \left\{ \int_{\mathcal{X}} \left(\partial^K \mu(x)\right)^2 w(x) dx \right\} \\ \times \left(\frac{2}{1+2K} - \left(\int_{-1}^{1} R(x) x^K dx \right)' \left(\int_{-1}^{1} R(x) R(x)' dx \right)^{-1} \left(\int_{-1}^{1} R(x) x^K dx \right) \right). \end{aligned}$$

Alternatively, if $[m] = 0$ and $K = 1$, we have

$$\mathscr{B}_{1,d,0} = \frac{1}{12} \sum_{\ell=1}^{d} |\mathcal{X}_\ell|^2 \int_{\mathcal{X}} \left(\frac{\partial \mu(x)}{\partial x_\ell} \right)^2 w(x) dx,$$

using $R_j(z) = 1$ and $[k] = [\tilde{k}] = 1$, so that $\int_{\mathcal{U}} R_j(z)' z^{\tilde{k}} dz = 0$, and further, if $k \neq \tilde{k}$, then $\int_{\mathcal{U}} z^{k+\tilde{k}} dz = 0$, whence the entire term in braces is zero; otherwise $k_\ell + \tilde{k}_\ell = 2$ and $2^{d-1} \int_{-1}^{1} z_\ell^2 dz_\ell = 2^d/3$. □

A.4 Bahadur Representation and Asymptotic Normality

For completeness, we give the explicit form of the random function $\nu_n(x)$ from Eqn. (3). It is the remainder in the Bahadur representation of the identity functional, $\theta_{1,0}$ of Example 1 or $\theta_{2,0}$ of Example 2. Recall the definition of $\psi_n(x, z)$ from the text and write: $\hat{\mu}(x) - \mu(x) =$

$\frac{1}{n}\sum_{i=1}^{n}\psi_n(x,X_i)\varepsilon_i+\nu_n(x)$, where the remainder $\nu_n(x)$ is given by:

$$\begin{aligned}\nu_n(x) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{J_n^d}&\tilde{R}_j(x)'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\hat{\Omega}_j^{-1}\tilde{R}_j(X_i)\varepsilon_i/q_j\\
&+\sum_{j=1}^{J_n^d}\tilde{R}_j(x)'\mathbb{1}_{n,j}\hat{\Omega}_j^{-1}\tilde{R}_j'(G-\tilde{R}_j\beta_j^0)/(nq_j)\\
&+\sum_{j=1}^{J_n^d}\mathbb{1}_{n,j}(\tilde{R}_j(x)'\beta_j^0-\mu_j(x))\\
&+\sum_{j=1}^{J_n^d}(\mathbb{1}_{n,j}-1)\left[\mu_j(x)+\tilde{R}_j(x)'\Omega_j^{-1}\tilde{R}_j'(Y-G)/(nq_j)\right].\end{aligned}$$

Proof of Theorem 4. Recall from the text that $\Theta_j=(\theta([R_j(\cdot)]_1),\ldots,\theta([R_j(\cdot)]_{\dim(R(\cdot))}))'$. Under the linearity condition on $\theta(\cdot)$ in Assumption 3, we can write the remainder $\theta(\nu_n)$ from Eqn. (3) as

$$\theta(\nu_n)=\sum_{j=1}^{J_n^d}\Theta_j'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\hat{\Omega}_j^{-1}\tilde{R}_j'(Y-G)/(nq_j) \tag{T_{n1}}$$

$$+\sum_{j=1}^{J_n^d}\Theta_j'\mathbb{1}_{n,j}\hat{\Omega}_j^{-1}\tilde{R}_j'(G-\tilde{R}_j\beta_j^0)/(nq_j) \tag{T_{n2}}$$

$$+\sum_{j=1}^{J_n^d}\mathbb{1}_{n,j}(\Theta_j'\beta_j^0-\theta(\mu_j)) \tag{T_{n3}}$$

$$+\sum_{j=1}^{J_n^d}(\mathbb{1}_{n,j}-1)\left[\theta(\mu_j)+\Theta_j'\Omega_j^{-1}\tilde{R}_j'(Y-G)/(nq_j)\right]. \tag{T_{n4}}$$

For T_{n1} write:

$$T_{n1}=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{J_n^d}\Theta_j'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\hat{\Omega}_j^{-1}\tilde{R}_j(X_i)\varepsilon_i/q_j \tag{T_{n11}}$$

$$-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{J_n^d}\Theta_j'\mathbb{1}_{n,j}\Omega_j^{-1}(\hat{\Omega}_j-\Omega_j)\Omega_j^{-1}\tilde{R}_j(X_i)\varepsilon_i/q_j. \tag{T_{n12}}$$

Applying linearity and then continuity of the functional $\theta(\cdot)$ from Assumption 3, followed by

Lemmas A.1, A.3, A.4, and A.5 we have the following bound on $|T_{n11}|$:

$$\begin{aligned}
|T_{n11}| &= \left|\theta\left(\sum_{j=1}^{J_n^d}(\tilde{R}_j(\cdot))'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\hat{\Omega}_j^{-1}\frac{\tilde{R}_j'(Y-G)}{nq_j}\right)\right| \\
&\leq C\max_{m:[m]\leq s}\left\|\sum_{j=1}^{J_n^d}(\partial^m\tilde{R}_j(\cdot))'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\Omega_j^{-1}(\Omega_j-\hat{\Omega}_j)\hat{\Omega}_j^{-1}\frac{\tilde{R}_j'(Y-G)}{nq_j}\right\|_\infty \\
&\leq C\left(\max_{1\leq j\leq J_n^d}\max_{m:[m]\leq s}\|\partial^m\tilde{R}_j(\cdot)\|_\infty\right)\left(\max_{1\leq j\leq J_n^d}\left|\Omega_j-\hat{\Omega}_j\right|^2\right)\left(\max_{1\leq j\leq J_n^d}\left|\mathbb{1}_{n,j}\hat{\Omega}_j^{-1}\right|\right) \\
&\qquad\times\left(\max_{1\leq j\leq J_n^d}\left|\Omega_j^{-1}\right|^2\right)\left(\max_{1\leq j\leq J_n^d}\left|\frac{\tilde{R}_j'(Y-G)}{nq_j}\right|\right) \\
&= O_p\left(J_n^s\frac{J_n^d\log(J_n^d)}{n}\frac{J_n^{d-d\xi/2}\log(J_n^d)^{\xi/2}}{\sqrt{n}}\right) \\
&= O_p\left(\frac{J_n^{(2-\xi/2)d+s}\log(J_n^d)^{1+\xi/2}}{n^{3/2}}\right).
\end{aligned}$$

For T_{n12}, begin by defining

$$W_j(i,l)=\mathbb{1}_{n,j}\Omega_j^{-1}\left(\tilde{R}_j(X_i)\tilde{R}_j(X_i)'-\mathbb{E}[\tilde{R}_j(X_i)\tilde{R}_j(X_i)']\right)\Omega_j^{-1}\tilde{R}_j(X_l)\varepsilon_l,$$

so that we can express T_{n12} as

$$T_{n12}=\sum_{j=1}^{J_n^d}\frac{1}{(nq_j)^2}\sum_{i=1}^{n}\sum_{l=1}^{n}\Theta_j'W_j(i,l).$$

Observe that $\mathbb{E}[T_{n12}]=0$ and that unless $i=h$ and $l=m$, $\mathbb{E}\left[W_j(i,l)W_j(h,m)\right]=0$. By Lemmas A.1 and A.3, Assumption 1(c), and $q_j\asymp J_n^{-d}$, we have:

$$\begin{aligned}
\max_{1\leq j\leq J_n^d}\mathbb{E}\left[W_j(i,i)W_j(i,i)'\right] &\leq C\left(\max_{1\leq j\leq J_n^d}\left|\Omega_j^{-1}\right|^4\right)\left(\left|\tilde{R}_j(\cdot)\right|_\infty^6\right)\left(\sup_{x\in\mathcal{X}}\sigma^2(x)\right)\max_{1\leq j\leq J_n^d}\mathbb{E}[\mathbb{1}_{P_j}(X_i)] \\
&= CO(1)O(1)\max_{1\leq j\leq J_n^d}q_j=O(J_n^{-d}), \qquad\qquad (A.11)
\end{aligned}$$

and similarly $\max_{1\leq j\leq J_n^d}\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right]=O(J_n^{-2d})$. Further note that Assumption 3 and Lemma A.1 give that:

$$\max_{1\leq j\leq J_n^d}|\Theta_j|\leq C\max_{1\leq j\leq J_n^d}\left(\max_{m:[m]\leq s}\|\partial^m\tilde{R}_j(\cdot)\|_\infty\right)=C\max_{1\leq j\leq J_n^d}\max_{m:[m]\leq s}\sup_{x\in P_j}|\partial^m\tilde{R}_j(x)|=O(J_n^s). \qquad (A.12)$$

Therefore the variance of T_{n2} may be bounded as follows, using $q_j \asymp J_n^{-d}$, Eqns. (A.11) and (A.12), linearity and continuity of $\theta(\cdot)$, and Lemma A.1:

$$\begin{aligned}
\mathbb{E}[T_{n2}^2] &= \sum_{j=1}^{J_n^d} \frac{1}{(nq_j)^4} \sum_{i=1}^{n}\sum_{l=1}^{n} \Theta_j' \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right]\Theta_j \\
&\leq \frac{CJ_n^{4d}}{n^4} \sum_{j=1}^{J_n^d} \Theta_j' \left\{ n\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1)\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right]\right\}\Theta_j \\
&= \frac{J_n^{4d}}{n^4}\theta\left(\sum_{j=1}^{J_n^d} \tilde{R}_j(\cdot)' \left\{ n\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1)\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right]\right\}\Theta_j \right) \\
&\leq \frac{CJ_n^{4d}}{n^4} \max_{m:[m]\leq s} \left\| \sum_{j=1}^{J_n^d} (\partial^m \tilde{R}_j(\cdot))' \left\{ n\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1)\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right]\right\}\Theta_j \right\|_\infty \\
&\leq \frac{CJ_n^{4d}}{n^4} \left(\max_{1\leq j\leq J_n^d} |\Theta_j| \right) \left(\max_{1\leq j\leq J_n^d} n\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1)\mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \right) \\
&\qquad \times \left(\max_{m:[m]\leq s} \max_{1\leq j\leq J_n^d} \sup_{x\in P_j} (\partial^m \tilde{R}_j(\cdot)) \right) \\
&= \frac{CJ_n^{4d}}{n^4} O(J_n^s) \left\{ nJ_n^{-d} + n^2 J_n^{-2d} \right\} O(J_n^s) \\
&= O_p\left(J_n^{2d+2s}/n^2 \right).
\end{aligned}$$

Hence $|T_{n2}| = O_p\left(J_n^{d+s}/n\right)$, by Markov's inequality.

Following similar logic as T_{n11}, by linearity, continuity, Lemmas A.1 and A.4, and Eqn. (A.5):

$$\begin{aligned}
|T_{n2}| &= \left| \theta\left(\sum_{j=1}^{J_n^d} \tilde{R}_j(\cdot)' \mathbb{1}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0)/(nq_j) \right) \right| \\
&\leq C \max_{m:[m]\leq s} \left\| \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0)/(nq_j) \right\|_\infty \\
&\leq C \max_{m:[m]\leq s} \max_{1\leq j\leq J_n^d} \sup_{x\in P_j} \left| \partial^m \tilde{R}_j(x)' \mathbb{1}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0)/(nq_j) \right| \\
&\leq \left(\max_{1\leq j\leq J_n^d} \max_{m:[m]\leq s} \|\partial^m \tilde{R}_j(\cdot)\|_\infty \right) \left(\max_{1\leq j\leq J_n^d} \left| \mathbb{1}_{n,j} \hat{\Omega}_j^{-1/2} \right| \right) \left(\max_{1\leq j\leq J_n^d} \left| \mathbb{1}_{n,j} \hat{\Omega}_j^{-1/2} \tilde{R}_j' \frac{(G - \tilde{R}_j \beta_j^0)}{nq_j} \right| \right) \\
&= O_p\left(J_n^{-((S+\alpha)\wedge K - s)} \right).
\end{aligned}$$

Next, similar logic gives:

$$
\begin{aligned}
|T_{n3}| &= \left| \theta \left(\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} (\tilde{R}_j(\cdot)'\beta_j^0 - \partial^m \mu_j(\cdot))) \right) \right| \\
&\leq C \max_{m:[m]\leq s} \left\| \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} (\tilde{R}_j(\cdot)'\beta_j^0 - \partial^m \mu_j(\cdot))) \right\|_\infty \\
&\leq C \max_{m:[m]\leq s} \max_{1\leq j\leq J_n^d} \left\| \tilde{R}_j(\cdot)'\beta_j^0 - \partial^m \mu_j(\cdot))) \right\|_\infty \\
&= O_p\left(J_n^{-((S+\alpha)\wedge K - s)} \right),
\end{aligned}
$$

directly by Lemma A.2. Finally, from $\min_{1\leq j\leq J_n^d} \mathbb{1}_{n,j} = 1$ w.p.a. 1 it follows that T_{n4} is smaller order than the other terms. This completes the proof. □

We now demonstrate a version of Theorem 4 that holds with probability one.

Theorem A.2. *Let Assumption 3 hold with $s \leq S \wedge (K-1)$, and consider the representation in Eqn.* (3)*. If the conditions of Theorem A.1 hold, then:*

$$
\theta(\nu_n) = O_{as}\left(\frac{J_n^{(3/2-\xi/2)d+s} \log(J_n^d)^{(1+\xi)/2}}{n} + J_n^{-((S+\alpha)\wedge K - s)} \right).
$$

Proof of Theorem A.2. Use the same expansion as in the proof of Theorem 4. Remainders T_{n2}, T_{n3}, and T_{n4} are handled identically, applying the almost sure versions of the same steps. For T_{n1} we use similar steps as above for T_{n11}. Applying linearity and then continuity of the functional $\theta(\cdot)$ from Assumption 3, followed by Lemmas A.1, A.3, A.4, and A.5 we have the following bound on $|T_{n1}|$:

$$
\begin{aligned}
|T_{n1}| &= \left| \theta \left(\sum_{j=1}^{J_n^d} (\tilde{R}_j(\cdot))'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j - \hat{\Omega}_j)\hat{\Omega}_j^{-1} \frac{\tilde{R}_j'(Y-G)}{nq_j} \right) \right| \\
&\leq C \max_{m:[m]\leq s} \left\| \sum_{j=1}^{J_n^d} (\partial^m \tilde{R}_j(\cdot))'\mathbb{1}_{n,j}\Omega_j^{-1}(\Omega_j - \hat{\Omega}_j)\hat{\Omega}_j^{-1} \frac{\tilde{R}_j'(Y-G)}{nq_j} \right\|_\infty \\
&\leq C \left(\max_{1\leq j\leq J_n^d} \max_{m:[m]\leq s} \|\partial^m \tilde{R}_j(\cdot)\|_\infty \right) \left(\max_{1\leq j\leq J_n^d} \left| \Omega_j - \hat{\Omega}_j \right| \right) \left(\max_{1\leq j\leq J_n^d} \left| \mathbb{1}_{n,j}\hat{\Omega}_j^{-1} \right| \right) \\
&\qquad \times \left(\max_{1\leq j\leq J_n^d} \left| \Omega_j^{-1} \right| \right) \left(\max_{1\leq j\leq J_n^d} \left| \frac{\tilde{R}_j'(Y-G)}{nq_j} \right| \right) \\
&= O_{as}\left(J_n^s \sqrt{\frac{J_n^d \log(J_n^d)}{n}} \frac{J_n^{d-d\xi/2} \log(J_n^d)^{\xi/2}}{\sqrt{n}} \right)
\end{aligned}
$$

$$= O_{as}\left(\frac{J_n^{d(3-\xi)/2+s}\log(J_n^d)^{(1+\xi)/2}}{n}\right),$$

where the second inequality holds because the functional only operates on $\tilde{R}_j(\cdot)$. □

Proof of Theorem 5(a). Recall the definitions given in Eqn. (4) and that consecutive uses of the symbol $\asymp$ are to be interpreted pairwise. By assumption $\sigma^2(x)$ is bounded away from zero on $\mathcal{X}$, so for some $\overline{\sigma}$, $\sigma^2(\cdot) \asymp \overline{\sigma}$. Then under Assumption 1(c) we have $\Gamma_j \asymp \overline{\sigma}\mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right] \asymp \Omega_j$. Again using $\sigma^2(\cdot) \asymp \overline{\sigma}$ and $\Gamma_j \asymp \Omega_j$, and further by $q_j \asymp J_n^{-d}$ and Lemma A.3 we have:

$$\begin{aligned} &V_n = \mathbb{E}\left[\Psi_n(X)^2\sigma^2(X)\right] \asymp \mathbb{E}[\Psi_n(X)^2] = \|\Psi_n\|_2^2, \text{ and also} \\ &V_n \asymp \sum_{j=1}^{J_n^d} \Theta_j'\Omega_j^{-1}\Theta_j/q_j \asymp J_n^d \sum_{j=1}^{J_n^d} |\Theta_j|^2. \end{aligned} \tag{A.13}$$

The condition that $\theta(\nu_n) = o_p(\sqrt{V_n}/\sqrt{n})$ and the result of Theorem 4 immediately give the triangular array representation of the Theorem. By construction, $\mathbb{E}\left[\Psi_n(X_i)\varepsilon_i/\sqrt{nV_n}\right] = 0$ and $\sum_{i=1}^n \mathbb{E}\left[\left(\Psi_n(X_i)\varepsilon_i/\sqrt{nV_n}\right)^2\right] = 1$. It remains to verify the Lindeberg condition. For any $\delta > 0$, by the Hölder and Markov inequalities, Assumption 1(c), $V_n \asymp \|\Psi_n\|_2^2$ by Eqn. (A.13), and the conditions of the Theorem,

$$\begin{aligned} \sum_{i=1}^n \mathbb{E}\left[\left(\frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV_n}}\right)^2 \mathbb{1}\left\{\left|\frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV_n}}\right| > \delta\right\}\right] &\leq n\left(\mathbb{E}\left[\left(\frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV_n}}\right)^{2+\eta}\right]\right)^{\frac{2}{2+\eta}} \left(\mathbb{P}\left[\left|\frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV_n}}\right| > \delta\right]\right)^{\frac{\eta}{2+\eta}} \\ &\leq \frac{n}{\delta^\eta}\mathbb{E}\left[\left|\frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV_n}}\right|^{2+\eta}\right] \\ &= \frac{1}{\delta^\eta}\frac{\mathbb{E}\left[|\Psi_n(X_i)|^{2+\eta}\mathbb{E}[|\varepsilon_i|^{2+\eta} \mid X_i]\right]}{n^{\eta/2}V_n^{1+\eta/2}} \\ &= O\left(\left(\frac{\|\Psi_n\|_{2+\eta}}{n^{\eta/(4+2\eta)}\|\Psi_n\|_2}\right)^{2+\eta}\right) \to 0. \end{aligned}$$

Convergence in distribution follows by the Lindeberg-Feller central limit theorem.

For the second conclusion of Theorem 5(a), observe that by $\mathbb{1}_{n,j} = 1$ w.p.a. 1, uniformly in j, we have $\hat{V}_n/V_n - 1 = T_{n1} + T_{n2} + T_{n3} + o_p(1)$, where

$$T_{n1} = V_n^{-1}\hat{V}_n - V_n^{-1}\sum_{j=1}^{J_n^d}\mathbb{1}_{n,j}\Theta_j'\hat{\Omega}_j^{-1}\tilde{\Gamma}_j\hat{\Omega}_j^{-1}\Theta_j/q_j,$$

$$T_{n2} = V_n^{-1}\sum_{j=1}^{J_n^d}\mathbb{1}_{n,j}\Theta_j'(\hat{\Omega}_j^{-1} + \Omega_j^{-1})\tilde{\Gamma}_j\left(\hat{\Omega}_j^{-1} - \Omega_j^{-1}\right)\Theta_j/q_j,$$

$$T_{n3} = V_n^{-1} \sum_{j=1}^{J_n^d} \Theta_j' \Omega_j^{-1} \left(\tilde{\Gamma}_j - \Gamma_j \right) \Omega_j^{-1} \Theta_j / q_j,$$

and $\tilde{\Gamma}_j = \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \varepsilon_i^2 / (nq_j)$. First, expanding the squared terms, T_{n1} can be split into two terms, and upon applying Lemmas A.1 and A.4, $q_j \asymp J_n^{-d}$, Eqns. (A.4) and (A.13), and the condition of the Theorem, we find that

$$\begin{aligned}
T_{n1} &= V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \Theta_j' \hat{\Omega}_j^{-1} \left(\frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' (\hat{\mu}(X_i) - \mu(X_i))^2 \right) \hat{\Omega}_j^{-1} \Theta_j / q_j \\
&\quad - V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \Theta_j' \hat{\Omega}_j^{-1} \left(\frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' 2\varepsilon_i (\hat{\mu}(X_i) - \mu(X_i)) \right) \hat{\Omega}_j^{-1} \Theta_j / q_j \\
&\leq \left(\max_{1 \leq j \leq J_n^d} \mathbb{1}_{n,j} |\hat{\Omega}_j^{-1}|^2 \right) \left(\max_{1 \leq j \leq J_n^d} \|\tilde{R}_j(\cdot)\|_\infty^2 \right) \left(\|\hat{\mu} - \mu\|_\infty \right) \\
&\quad \times \left\{ \|\hat{\mu} - \mu\|_\infty \frac{J_n^d}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 \frac{1}{nq_j} \sum_{i=1}^n \mathbb{1}_{P_j}(X_i) + \frac{J_n^d}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 \frac{1}{nq_j} \sum_{i=1}^n \mathbb{1}_{P_j}(X_i) |\varepsilon_i| \right\} \\
&= O_p\left(\|\hat{\mu} - \mu\|_\infty \right) \times \left\{ o_p(1) O(1) O_p(1) + O_p(1) \right\} = o_p(1),
\end{aligned}$$

where the final line additionally uses Assumption 1(c) and the final relation of Eqn. (A.13) to give:

$$\mathbb{E}\left[\frac{J_n^d}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 \frac{1}{nq_j} \sum_{i=1}^n \mathbb{1}_{P_j}(X_i) |\varepsilon_i| \right] \leq C \frac{J_n^d}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 \frac{\mathbb{E}\left[\mathbb{1}_{P_j}(X_i) \mathbb{E}\left[|\varepsilon_i| \mid X_i \right] \right]}{q_j} = O(1).$$

By Lemma A.1 and otherwise identical steps to the above, we get:

$$\mathbb{E}\left[\frac{1}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 |\tilde{\Gamma}_j| / q_j \right] \leq \frac{J_n^d}{V_n} \sum_{j=1}^{J_n^d} |\Theta_j|^2 \frac{1}{nq_j} \sum_{i=1}^n \mathbb{E}\left[\left| \tilde{R}_j(X) \right|^2 \varepsilon_i^2 \right] = O(1).$$

Therefore, applying Lemmas A.3 and A.4:

$$\begin{aligned}
|T_{n2}| &= V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{1}_{n,j} \Theta_j' (\hat{\Omega}_j^{-1} + \Omega_j^{-1}) \tilde{\Gamma}_j \Omega_j^{-1} \left(\hat{\Omega}_j - \Omega_j \right) \hat{\Omega}_j^{-1} \Theta_j / q_j \\
&\leq C \left(\max_{1 \leq j \leq J_n^d} \mathbb{1}_{n,j} |\hat{\Omega}_j^{-1}|^3 \vee \max_{1 \leq j \leq J_n^d} |\Omega_j^{-1}|^3 \right) \left(\max_{1 \leq j \leq J_n^d} |\hat{\Omega}_j - \Omega_j| \right) V_n^{-1} \sum_{j=1}^{J_n^d} |\Theta_j|^2 |\tilde{\Gamma}_j| / q_j \\
&= O_p\left(\sqrt{J_n^d \log(J_n^d)/n} \right) = o_p(1).
\end{aligned}$$

Finally, referring to the definitions in Eqn. (3), observe that $T_{n3} = \sum_{i=1}^{n} T_{n3}(i)/n$, where $T_{n3}(i) = V_n^{-1}(\Psi_n(X_i)^2\varepsilon_i^2 - \mathbb{E}[\Psi_n(X_i)^2\varepsilon_i^2])$, so that $\mathbb{E}[T_{n3}(i)] = 0$. Consider two cases. First, suppose $\eta < 2$. Then by Burkholder's inequality, the fact that for $\delta \in (0,1)$, $(a+b)^{(1+\delta)/2} \leq a^{(1+\delta)/2} + b^{(1+\delta)/2}$, the c_r inequality, Jensen's inequality, Assumption 1(c), and the first relation of Eqn. (A.13):

$$\begin{aligned}
\mathbb{E}\left[\left|\frac{1}{n}\sum_{i=1}^{n} T_{n3}(i)\right|^{1+\eta/2}\right] &\leq \frac{C}{n^{1+\eta/2}}\mathbb{E}\left[\left|\sum_{i=1}^{n} T_{n3}(i)^2\right|^{(1+\eta/2)/2}\right] \\
&\leq \frac{C}{n^{1+\eta/2}}\mathbb{E}\left[\sum_{i=1}^{n} |T_{n3}(i)|^{1+\eta/2}\right] \\
&\leq \frac{C}{n^{\eta/2}} 2^{\eta/2} \frac{\mathbb{E}\left[\left|\Psi_n(X_i)^2\varepsilon_i^2\right|^{1+\eta/2}\right] + \left(\mathbb{E}\left[\Psi_n(X_i)^2\varepsilon_i^2\right]\right)^{1+\eta/2}}{V_n^{1+\eta/2}} \\
&\leq \frac{C}{n^{\eta/2}} \frac{\mathbb{E}\left[|\Psi_n(X_i)|^{2+\eta}\,\mathbb{E}\left[|\varepsilon_i|^{2+\eta} \mid X\right]\right] + \left(\mathbb{E}\left[\Psi_n(X_i)^2\sigma^2(X)\right]\right)^{1+\eta/2}}{V_n^{1+\eta/2}} \\
&= O\left(\left(\frac{\|\Psi_n\|_{2+\eta}}{n^{\eta/(4+2\eta)}\|\Psi_n\|_2}\right)^{2+\eta}\right) \to 0.
\end{aligned}$$

Next, for the case of $\eta \geq 2$ we utilize only the fourth moment to find that:

$$\begin{aligned}
\mathbb{E}\left[\left(\frac{1}{n}\sum_{i=1}^{n} T_{n3}(i)\right)^2\right] &= \frac{1}{n}\mathbb{E}\left[T_{n3}(i)^2\right] = \frac{1}{n}\mathbb{E}\left[V_n^{-1}(\Psi_n(X_i)^2\varepsilon_i^2 - \mathbb{E}[\Psi_n(X_i)^2\varepsilon_i^2])^2\right] \\
&\leq \frac{1}{n}V_n^{-2}\mathbb{E}\left[\Psi_n(X_i)^4\varepsilon_i^4\right] \\
&= O\left(\left(\frac{\|\Psi_n\|_4}{n^{1/4}\|\Psi_n\|_2}\right)^4\right) \to 0,
\end{aligned}$$

again using Jensen's inequality, Assumption 1(c), and the first relation of Eqn. (A.13). In either case, $T_{3n} = o_p(1)$ by Markov's inequality. □

Proof of Theorem 5(b). By Assumption 1(c), the Cauchy-Schwarz and triangle inequalities, and the conditions of the Theorem:

$$\begin{aligned}
V_n - V &= \mathbb{E}[(\Psi_n(X)^2 - \Psi(X)^2)\sigma^2(X)] \\
&= \mathbb{E}[(\Psi_n(X) - \Psi(X))(\Psi_n(X) + \Psi(X))\sigma^2(X)] \\
&= \mathbb{E}[(\Psi_n(X) - \Psi(X))(\Psi_n(X) - \Psi(X) + 2\Psi(X))\sigma^2(X)] \\
&\leq C\mathbb{E}[(\Psi_n(X) - \Psi(X))^2]^{1/2}\mathbb{E}[(\Psi_n(X) - \Psi(X) + 2\Psi(X))^2]^{1/2}
\end{aligned}$$

$$= C\|\Psi_n - \Psi\|_2(\|\Psi_n - \Psi + 2\Psi\|_2)$$
$$\leq C\|\Psi_n - \Psi\|_2(\|\Psi_n - \Psi\|_2 + 2\|\Psi\|_2) \to 0, \tag{A.14}$$

whence the second conclusion.

Using the above result, the assumed mean-square convergence of $\Psi_n(X)$, and the remainder condition of the Theorem,

$$\frac{\sqrt{n}(\theta(\hat{\mu}) - \theta(\mu))}{\sqrt{V_n}} = \sum_{i=1}^{n}\left[\frac{\Psi(X_i)\varepsilon_i}{\sqrt{nV}} + \frac{(\Psi_n(X_i) - \Psi(X_i))\varepsilon_i}{\sqrt{nV}} + \frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV}}\left(\frac{\sqrt{V}}{\sqrt{V_n}} - 1\right)\right] + \frac{\sqrt{n}\theta(\nu_n)}{\sqrt{V_n}}$$
$$= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\Psi(X_i)\varepsilon_i}{\sqrt{V}} + o_p(1).$$

Convergence in distribution now follows under the assumed moment condition on $\Psi(X)$ and a standard central limit theorem.

For the final conclusion, as in the proof of Theorem 5(a) write $\hat{V}_n/V_n - 1 = T_{n1} + T_{n2} + T_{n3} + o_p(1)$, for T_{n1}, T_{n2}, and T_{n3} defined there. As above, $T_{n1} = o_p(1)$ and $T_{n2} = o_p(1)$. Next,

$$T_{n3} = \left(\frac{1}{V_n} - \frac{1}{V}\right)\frac{1}{n}\sum_{i=1}^{n}\Psi_n(X_i)^2\varepsilon_i^2 + \frac{1}{n}\sum_{i=1}^{n}\frac{[\Psi_n(X_i)^2 - \Psi(X_i)^2]\varepsilon_i^2}{V} + \frac{1}{nV}\sum_{i=1}^{n}(\Psi(X_i)^2\varepsilon_i^2 - V),$$

where the first two terms tend to zero in probability by Eqn. (A.14) (and the steps therein) and Markov's inequality, and the third by the law of large numbers. □

B Unconditional IMSE Expansion for $K = 1$

In the especial case of a piecewise constant fit ($K = 1$) the leading constants in the unconditional IMSE may also be computed explicitly. The special structure of the constant-fit partitioning estimator is crucial to obtaining this result. When $K = 1$, $R_j'R_j = N_j$ is a binomial random variable whose inverse moments can be calculated or approximated accurately, as done in Lemma B.1 below. The Theorem below shows that for this special case, the unconditional IMSE has the same leading constants as the conditional IMSE.

Theorem B.1. *Suppose the conditions of Theorem 3 hold with $S = 0$. Then, if $w(x)$ is continuous, the piecewise-constant partitioning estimator ($K = 1$) satisfies:*

$$\int_{\mathcal{X}} \mathbb{E}[(\hat{\mu}(x) - \mu(x))^2] w(x) dx = \frac{J_n^d}{n}[\mathscr{V}_{1,d,0} + o(1)] + \frac{1}{J_n^2}[\mathscr{B}_{1,d,0} + o(1)].$$

Prior to proving the Theorem, we give several results regarding binomial random variables.

Lemma B.1. *Let the conditions of Theorem B.1 hold. Recall that for $K = 1$, $\tilde{R}_j'\tilde{R}_j = \sum_{i=1}^n \mathbb{1}_{P_j}(X_i)$ is the number of observations in P_j. Call this N_j. Further define $N_{j,-i} = \sum_{l \neq i} \mathbb{1}_{P_j}(X_l)$ and $N_{j,-i-l} = \sum_{m \neq i,l} \mathbb{1}_{P_j}(X_m)$. Then $N_j \sim Bin(n, q_j)$, $N_{j,-i} \sim Bin(n-1, q_j)$, and $N_{j,-i-l} \sim Bin(n-2, q_j)$. All remainder terms are uniform in $1 \leq j \leq J_n^d$.*

1. $\mathbb{E}\left[\mathbb{1}\{N_j > 0\}\frac{1}{N_j}\right] = \frac{1}{nq_j} + o\left(J_n^d/n\right).$
2. $\mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] = \frac{1-(1-q_j)^n}{nq_j}.$
3. $\mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^2}\right] = \frac{1}{(nq_j)^2}\left(1 + o\left(J_n^d/n\right)\right)$
4. $\mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^2}\right] = \frac{1}{n(n-1)q_j^2}\left(1 - \frac{1}{nq_j}\left(1 + o\left(J_n^d/n\right)\right)\right).$

Proof. The first result follows from Rempala (2003, Proceeds of the American Mathematical Society) or Znidaric (2009, The Open Statistics and Probability Journal), whose expansions remain valid if $q_j \to 0$, $nq_j \to \infty$, and because $q_j \asymp J_n^{-d}$, the result holds uniformly in $1 \leq j \leq J_n^d$. The final three results are proven by direct calculation: an exact expression for each moment may be found in terms of n, q_j, and $\mathbb{E}[\mathbb{1}\{N_j > 0\}N_j^{-1}]$, and then the claims follow by substituting the first result. The calculations are as follows, where we make use of the facts that

$$\frac{n}{k}\binom{n-1}{k-1} = \binom{n}{k}, \quad \text{and,} \quad \mathbb{E}\left[\mathbb{1}\{N_j > 0\}\frac{1}{N_j}\right] = \sum_{k=1}^{n} \frac{1}{k}\binom{n}{k} q_j^k (1-q_j)^{n-k}.$$

For the second result:

$$
\begin{aligned}
\mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] &= \sum_{k=0}^{n-1} \frac{1}{k+1}\binom{n-1}{k} q_j^k (1-q_j)^{n-1-k} \\
&= \sum_{\tilde{k}=1}^{n} \frac{1}{\tilde{k}}\binom{n-1}{\tilde{k}-1} q_j^{\tilde{k}-1}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{nq_j}\sum_{\tilde{k}=1}^{n}\binom{n}{\tilde{k}} q_j^{\tilde{k}}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{nq_j}\mathbb{P}[N_j>0] = \frac{1}{nq_j}\left(1-\mathbb{P}[N_j=0]\right) \\
&= \frac{1-(1-q_j)^n}{nq_j}.
\end{aligned}
$$

For the third result:

$$
\begin{aligned}
\mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^2}\right] &= \sum_{k=0}^{n-1} \frac{1}{(k+1)^2}\binom{n-1}{k} q_j^k (1-q_j)^{n-1-k} \\
&= \sum_{\tilde{k}=1}^{n} \frac{1}{\tilde{k}^2}\binom{n-1}{\tilde{k}-1} q_j^{\tilde{k}-1}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{nq_j}\sum_{\tilde{k}=1}^{n}\frac{1}{\tilde{k}}\binom{n}{\tilde{k}} q_j^{\tilde{k}}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{nq_j}\mathbb{E}\left[\mathbb{1}\{N_j>0\}\frac{1}{N_j}\right].
\end{aligned}
$$

For the fourth result:

$$
\begin{aligned}
\mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^2}\right] &= \sum_{k=0}^{n-2} \frac{1}{(k+2)^2}\binom{n-2}{k} q_j^k (1-q_j)^{n-2-k} \\
&= \sum_{\tilde{k}=2}^{n} \frac{1}{\tilde{k}^2}\binom{n-2}{\tilde{k}-2} q_j^{\tilde{k}-2}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{n(n-1)q_j^2}\sum_{\tilde{k}=1}^{n}\frac{\tilde{k}-1}{\tilde{k}}\binom{n}{\tilde{k}} q_j^{\tilde{k}}(1-q_j)^{n-\tilde{k}} \\
&= \frac{1}{n(n-1)q_j^2}\left\{\sum_{\tilde{k}=1}^{n}\binom{n}{\tilde{k}} q_j^{\tilde{k}}(1-q_j)^{n-\tilde{k}} - \sum_{\tilde{k}=1}^{n}\frac{1}{\tilde{k}}\binom{n}{\tilde{k}} q_j^{\tilde{k}}(1-q_j)^{n-\tilde{k}}\right\} \\
&= \frac{1}{n(n-1)q_j^2}\left(\mathbb{E}\left[\mathbb{1}\{N_j>0\}\right] - \mathbb{E}\left[\mathbb{1}\{N_j>0\}\frac{1}{N_j}\right]\right)
\end{aligned}
$$

$$= \frac{1}{n(n-1)q_j^2}\left(1 - (1-q_j)^n - \mathbb{E}\left[\mathbb{1}\{N_j > 0\}\frac{1}{N_j}\right]\right)$$
$$= \frac{1}{n(n-1)q_j^2}\left(1 - \mathbb{E}\left[\mathbb{1}\{N_j > 0\}\frac{1}{N_j}\right]\right) - o(J_n^{2d}/n^2),$$

where the final equality uses $q_j \asymp J_n^{-d}$, hence $(1-q_j)^n \to 0$ exponentially. □

Proof of Theorem B.1. Recall that $\mathbf{X}_n = (X_1, \dots, X_n)'$ and expand as follows:

$$\int_X \mathbb{E}\left[(\hat{\mu}(x) - \mu(x))^2\right] f(x)dx = \int_{\mathcal{X}} \left\{\mathbb{E}\left[\mathbb{V}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] + \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] + \left(\mathbb{E}\left[\hat{\mu}(x)\right] - \mu(x)\right)^2\right\} f(x)dx.$$

We examine each term one at a time. The following two results will be used frequently. Recall that the volume of cell P_j is denoted $\mathrm{vol}\,(P_j)$ and similarly for $\mathrm{vol}\,(\mathcal{X})$. First observe that $q_j = \int_{P_j} f(z)dz = f(\overline{p}_j)\,\mathrm{vol}\,(P_j) + o(J_n^{-d})$, by Lemma A.6. Further, under the conditions placed on the partition and Assumption 1(b), $\mathrm{vol}\,(P_j) = \mathrm{vol}\,(\mathcal{X})\,/J_n^d$.

Consider the first term. Using the two results above, as well as Lemmas B.1, A.6, and A.7 (under Assumption 2(a)), we have:

$$\int_{\mathcal{X}} \mathbb{E}\left[\mathbb{V}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] w(x)dx = \int_{\mathcal{X}} \mathbb{E}\left[\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j}\mathbb{1}_{P_j}(x)\frac{1}{N_j^2}\sum_{i=1}^{n}\mathbb{1}_{P_j}(X_i)\sigma^2(X_i)\right] w(x)dx$$
$$= \int_{\mathcal{X}} \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \sum_{i=1}^{n} \mathbb{E}\left[\mathbb{1}_{n,j}\frac{1}{N_j^2}\mathbb{1}_{P_j}(X_i)\sigma^2(X_i)\right] w(x)dx$$
$$= \int_{\mathcal{X}} \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \sum_{i=1}^{n} \mathbb{E}\left[\frac{\mathbb{1}\{N_{j,-i} + \mathbb{1}_{P_j}(X_i) > 0\}}{(N_{j,-i} + \mathbb{1}_{P_j}(X_i))^2}\right] \mathbb{E}\left[\mathbb{1}_{P_j}(X_i)\sigma^2(X_i)\right] w(x)dx$$
$$= \sum_{j=1}^{J_n^d} \left(\int_{\mathcal{X}} \mathbb{1}_{P_j}(x)w(x)dx\right) \sum_{i=1}^{n} \mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^2}\right] \mathbb{E}\left[\mathbb{1}_{P_j}(X_i)\sigma^2(X_i)\right]$$
$$= \sum_{j=1}^{J_n^d} n\left(w(\overline{p}_j)\,\mathrm{vol}\,(P_j) + o(J_n^{-d})\right)\left(\frac{1}{(nq_j)^2} + o\left((J_n^d/n)^3\right)\right)\left(\int_{P_j} \sigma^2(z)f(z)dz\right)$$
$$= \sum_{j=1}^{J_n^d} nw(\overline{p}_j)\,\mathrm{vol}\,(P_j)\left(\frac{1}{n^2 q_j)^2} + o\left((J_n^d/n)^2\right)\right)\left(\sigma^2(\overline{p}_j)f(\overline{p}_j)\,\mathrm{vol}\,(P_j) + o(J_n^{-d})\right)$$
$$= \sum_{j=1}^{J_n^d} \frac{\sigma^2(\overline{p}_j)f(\overline{p}_j)w(\overline{p}_j)\,\mathrm{vol}\,(P_j)^2}{nq_j} + o\left(\frac{J_n^d}{n}\right)$$
$$= \frac{J_n^d}{\mathrm{vol}\,(\mathcal{X})\,n}\sum_{j=1}^{J_n^d} \frac{\sigma^2(\overline{p}_j)w(\overline{p}_j)}{f(\overline{p}_j)}\,\mathrm{vol}\,(P_j) + o\left(\frac{J_n^d}{n}\right)$$

$$= \frac{J_n^d}{\mathrm{vol}(\mathcal{X})\, n} \int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x)dx[1+o(1)] + o\left(J_n^d/n\right). \tag{B.1}$$

For the second variance term, define the following:

$$\bar{w}_j = \int_{P_j} w(x)dx; \qquad \bar{\mu}_j \equiv \mathbb{E}\left[\mathbb{1}_{P_j}(X)\mu(X)\right]; \qquad \bar{N}_j \equiv \mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] = \frac{1-(1-q_j)^n}{nq_j}.$$

Then we have:

$$\begin{aligned}
\int_{\mathcal{X}} \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] w(x)dx &= \int_{\mathcal{X}} \mathbb{V}\left[\sum_{j=1}^{J_n^d} \mathbb{1}_{n,j}\mathbb{1}_{P_j}(x) \sum_{i=1}^{n} \mathbb{1}_{P_j}(X_i)\mu(X_i)/N_j\right] w(x)dx \\
&= \int_{\mathcal{X}} \mathbb{E}\left[\sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \left\{\sum_{i=1}^{n} \frac{\mathbb{1}\{N_{j,-i}+1>0\}}{N_{j,-i}+1} \mathbb{1}_{P_j}(X_i)\mu(X_i) - \bar{\mu}_j\bar{N}_j\right\}^2\right] w(x)dx \\
&= \mathbb{E}\left[\sum_{j=1}^{J_n^d} \bar{w}_j \left\{\sum_{i=1}^{n} \frac{\mathbb{1}\{N_{j,-i}+1>0\}}{N_{j,-i}+1} \mathbb{1}_{P_j}(X_i)\mu(X_i) - \bar{\mu}_j\bar{N}_j\right\}^2\right] \\
&= \sum_{j=1}^{J_n^d} \bar{w}_j n\mathbb{E}\left[\mathbb{1}_{P_j}(X)\mu(X)^2\right] \mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^2}\right] && (V_{n1}) \\
&\quad + \sum_{j=1}^{J_n^d} \bar{w}_j n(n-1)\bar{\mu}_j^2 \mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^2}\right] && (V_{n2}) \\
&\quad - \sum_{j=1}^{J_n^d} \bar{w}_j (n\bar{\mu}_j\bar{N}_j)^2. && (V_{n3})
\end{aligned}$$

Now apply the two results above, as well as Lemmas B.1, A.6, and A.7 (under Assumption 2(a)), to get:

$$\begin{aligned}
V_{1n} &= \sum_{j=1}^{J_n^d} n\bar{w}_j \left(\int_{P_j} \mu(z)^2 f(z)dz\right) \frac{1}{(nq_j)^2}\left(1+o\left(J_n^d/n\right)\right) \\
&= \sum_{j=1}^{J_n^d} \bar{w}_j \left(\mu(\overline{p}_j)^2 q_j + o(J_n^{-d})\right)\left(\frac{1}{nq_j^2} + o\left(\left(J_n^d/n\right)^2\right)\right) \\
&= \frac{1}{n}\sum_{j=1}^{J_n^d} \frac{\mu(\overline{p}_j)^2\bar{w}_j}{q_j} + o\left(\frac{J_n^d}{n}\right). && \text{(B.2)}
\end{aligned}$$

Similarly:

$$
\begin{aligned}
V_{2n} &= \sum_{j=1}^{J_n^d} \bar{w}_j \bar{\mu}_j^2 n(n-1) \frac{1}{n(n-1)q_j^2} \left(1 - \frac{1}{nq_j}\left(1 + o\left(J_n^d/n\right)\right)\right) \\
&= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^3} + o\left(J_n^d/n\right) \\
&= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j (\mu(\overline{p}_j) q_j + o(J_n^{-d}))^2}{q_j^3} + o\left(J_n^d/n\right) \\
&= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \mu(\overline{p}_j)^2}{q_j} + o\left(J_n^d/n\right). \qquad \text{(B.3)}
\end{aligned}
$$

For the final term, similar steps give:

$$
\begin{aligned}
V_{3n} &= -\sum_{j=1}^{J_n^d} \bar{w}_j \bar{\mu}_j^2 n^2 \left(\frac{1-(1-q_j)^n}{nq_j}\right)^2 \\
&= -\sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} + O((1-q_j)^n). \qquad \text{(B.4)}
\end{aligned}
$$

Adding together Eqns. (B.2), (B.3), and (B.4) shows that

$$
\int_{\mathcal{X}} \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] w(x) dx = V_{1n} + V_{2n} + V_{3n} = o(J_n^d/n). \qquad \text{(B.5)}
$$

Finally, for the bias term, we first compute $\mathbb{E}\left[\hat{\mu}(x)\right]$ using Lemmas B.1 and A.6.

$$
\begin{aligned}
\mathbb{E}\left[\hat{\mu}(x)\right] &= \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \mathbb{E}\left[\mathbb{1}_{n,j} \frac{1}{N_j} \sum_{i=1}^{n} \mathbb{1}_{P_j}(X_i) \mu(X_i)\right] \\
&= \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] n \mathbb{E}\left[\mathbb{1}_{P_j}(X_i) \mu(X_i)\right] \\
&= \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \frac{1-(1-q_j)^n}{q_j} \int_{P_j} \mu(z) f(z) dz \\
&= \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x) \frac{\mu(\overline{p}_j) f(\overline{p}_j) \operatorname{vol}(P_j)}{f(\overline{p}_j) \operatorname{vol}(P_j)} \left(1 + o(J_n^{-d})\right)
\end{aligned}
$$

$$= \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x)\mu(\overline{p}_j)\left(1 + o(J_n^{-d})\right),$$

where all remainder terms are uniform in $1 \leq j \leq J_n^d$ and $x \in P_j$. By Assumption 2(b), with $S = 0$, $\mu(x)$ satisfies the Taylor expansion, with uniform (in j) remainder:

$$\mu(x) = \mu(\overline{p}_j) + \sum_{\ell=1}^{d} \frac{\partial \mu(\overline{p}_j)}{\partial x_\ell}(x_\ell - \overline{p}_{\ell,j}) + o(J_n^{-1}).$$

Next, by symmetry $(p_{\ell,j} - \overline{p}_{\ell,j}) = -(p_{\ell,j-1} - \overline{p}_{\ell,j}) = (p_{\ell,j} - p_{\ell,j-1})/2 = |\mathcal{X}_\ell|/(2J_n)$. Furthermore, Assumption 1(b) and symmetry, imply that $m \neq \ell$:

$$\int_{P_j} (x_\ell - \overline{p}_{\ell,j})(x_m - \overline{p}_{m,j})dx = 0, \text{ and,}$$

$$\int_{P_j} (x_\ell - \overline{p}_{\ell,j})^2 dx = \left(\prod_{\ell \neq m}^{d} (p_{\ell,j} - p_{\ell,j-1})\right)\left((p_{m,j} - \overline{p}_{m,j})^3 - (p_{m,j-1} - \overline{p}_{m,j})^3\right)/3 = J_n^{-2}\frac{1}{12}\operatorname{vol}(P_j)\,|\mathcal{X}_\ell|^2.$$

Then we have, using the above results and applying Assumption 2(a) and Lemma A.7:

$$\begin{aligned}
\int_{\mathcal{X}} \left(\mathbb{E}\left[\hat{\mu}(x)\right] - \mu(x)\right)^2 f(x)dx &= \int_{\mathcal{X}} \sum_{j=1}^{J_n^d} \mathbb{1}_{P_j}(x)\left(\mu(\overline{p}_j) - \mu(x)\right)^2 w(x)dx\left(1 + o(J_n^{-d})\right) \\
&= \sum_{j=1}^{J_n^d} \int_{P_j} \left(\sum_{\ell=1}^{d} \frac{\partial \mu(\overline{p}_j)}{\partial x_\ell}(x_\ell - \overline{p}_{\ell,j}) + o(J_n^{-1})\right)^2 w(x)dx\left(1 + o(J_n^{-d})\right) \\
&= \sum_{j=1}^{J_n^d} \int_{P_j} \left(\sum_{\ell=1}^{d} \frac{\partial \mu(\overline{p}_j)}{\partial x_\ell}(x_\ell - \overline{p}_{\ell,j}) + o(J_n^{-1})\right)^2 \\
&\qquad \times \left(w(\overline{p}_j) + (w(x) - w(\overline{p}_j))\right)dx\left(1 + o(J_n^{-d})\right) \\
&= \sum_{j=1}^{J_n^d} w(\overline{p}_j) \sum_{\ell=1}^{d} \left(\frac{\partial \mu(\overline{p}_j)}{\partial x_\ell}\right)^2 \int_{P_j} (x_\ell - \overline{p}_{\ell,j})^2 dx + o(J_n^{-2}) \\
&= J_n^{-2}\frac{1}{12} \sum_{j=1}^{J_n^d} w(\overline{p}_j) \sum_{\ell=1}^{d} |\mathcal{X}_\ell|^2 \left(\frac{\partial \mu(\overline{p}_j)}{\partial x_\ell}\right)^2 \operatorname{vol}(P_j) + o(J_n^{-2}) \\
&= J_n^{-2}\frac{1}{12} \sum_{\ell=1}^{d} |\mathcal{X}_\ell|^2 \int_{\mathcal{X}} \left(\frac{\partial \mu(x)}{\partial x_\ell}\right)^2 w(x)dx[1 + o(1)] + o(J_n^{-2}). \qquad \text{(B.6)}
\end{aligned}$$

Adding Eqns. (B.1), (B.5), and (B.6) gives the result. □

C Complete Simulation Results

This section contains the results from an exhaustive simulation study. We vastly extend the study contained in Section 5, but keep the aims and set up broadly the same; indeed the results from the main text are a subset of those given here. As such, herein we detail only the extensions considered, referring the reader to Section 5 for the for the general set up, tuning parameter choice, and description of the tables. Further, we do not attempt to interpret the many results presented beyond what is already discussed in the text.

Relative to Section 5, four extensions are presented. First, we consider univariate and trivariate data, in addition to the bivariate study.[1] For $d = 1$ and $d = 3$, we also choose appropriate points to examine boundary issues. Second, we also allow for a noisier model, setting $\sigma^2 = 4$, in addition to the unit variance. Third, we add the case where $X_{i,\ell} \sim \text{Beta}(2,2)$ (still truncated). Finally, keeping the tuning parameter J_n fixed, we place the cell boundaries (and knots) at the appropriate quantiles of the data. This is not technically covered by the theory, but is interesting with an eye toward applications. The regression functions considered are detailed below. Univariate results are presented in Section C.1, followed by complete bivariate results in C.2, and lastly Section C.3. contains results for $d = 3$.

For $d = 1$, we use the following specifications for $\mu(x)$. Models 1.1, 1.2, 1.3, and 1.7 are taken from Fan and Gijbels [1996. Local polynomial modelling and its applications. Chapter 4] and Models 1.4, 1.5, and 1.6 are adapted from Braun and Huang [2005. Kernel Spline Regression. The Canadian Journal of Statistics 33, 259–278]. Model 5 is altered to be discontinuous, not only nondifferentiable, whereas Model 6 has been smoothed. For Models 1.4 and 1.5, we set $J_n^* = 5$ in infeasible estimation. These regression functions are plotted in Figure C.1.

Model 1.1: $\mu(x) = \sin(8x - 4) + 2\exp\{-256(x - 1/2)^2\}$

Model 1.2: $\mu(x) = 4x - 2 + 2\exp\{-256(x - 1/2)^2\}$

Model 1.3: $\mu(x) = 8x/5 + 1/5$

Model 1.4: $\mu(x) = \mathbb{1}\{|4x - 2| < 0.1\}(1 - 100(4x - 2)^2)^4$

Model 1.5:
$$\begin{aligned}\mu(x) = {} & \mathbb{1}\{(4x-2) \in [-2,1]\}((4x-2)^7 - 19)/20 \\ & - \mathbb{1}\{(4x-2) \in (1,0]\}(4x-2)^2 \\ & + \mathbb{1}\{(4x-2) \in (0,1/2]\}(4x-2)^4/2 \\ & + \mathbb{1}\{(4x-2) \in (1/2,1]\}(4x-2)^5 \\ & + \mathbb{1}\{(4x-2) \in (1,2]\}(2 - (4x-2)^3)\end{aligned}$$

Model 1.6: $\mu(x) = (1 - (4x - 2)^2)^4$

[1] For $d = 3$, we perform 1,000 replications instead of 5,000, due to computational limitations.

Model 1.7: $\mu(x) = 0.3\exp\{-64(x-1/4)^2\} + 0.7\exp\{-256(x-1/2)^2\}$

Model 1.8: $\mu(x) = (x-1/2) + 8(x-1/2)^2 + 6(x-1/2)^3 - 30(x-1/2)^4 - 30(x-1/2)^5$

The first four bivariate specifications are as in Section 5. To these, we add four further models. Model 2.5 is taken from Fan and Gijbels [1996. Local polynomial modelling and its applications. Chapter 7]. All are plotted in Figure C.2.

$$\begin{aligned}
\text{Model 2.1: } \mu(x_1,x_2) &= 0.7\exp\left\{-3\left((4x_1-2+0.8)^2+8(x_2-1/2)^2\right)\right\} \\
&\qquad + \exp\left\{-3\left((4x_1-2-0.8)^2+8(x_2-1/2)^2\right)\right\} \\
\text{Model 2.2: } \mu(x_1,x_2) &= \sin(5x_1)\sin(10x_2) \\
\text{Model 2.3: } \mu(x_1,x_2) &= \left((1-(4x_1-2)^2)^2\right)(\sin(5x_2)/5) \\
\text{Model 2.4: } \mu(x_1,x_2) &= \mathbb{1}\{(4x_1-2)\in[-2,1]\}((4x_1-2)^7-19)/20 \\
&\quad - \mathbb{1}\{(4x_1-2)\in(-1,0]\}(4x_1-2)^2 \\
&\quad + \mathbb{1}\{(4x_1-2)\in(0,1/2]\}(4x_1-2)^4/2 \\
&\quad + \mathbb{1}\{(4x_1-2)\in(1/2,1]\}(4x_1-2)^5 \\
&\quad + \mathbb{1}\{(4x_1-2)\in(1,2]\}(2-(4x_1-2)^3) \\
&\quad + 4.26\left(\exp(-3x_2)-4\exp(-6x_2)+3\exp(-9x_2)\right) \\
&= \text{Model 1.5}(x_1) + 4.26\left(\exp(-3x_2)-4\exp(-6x_2)+3\exp(-9x_2)\right) \\
\text{Model 2.5: } \mu(x_1,x_2) &= (5/\pi)\exp\left\{-5(4x_1-2)^2/8\right\} \\
\text{Model 2.6: } \mu(x_1,x_2) &= \text{Model 1.3}(x_1) + \text{Model 1.3}(x_2) \\
\text{Model 2.7: } \mu(x_1,x_2) &= (\text{Model 1.3}(x_1))\,(\text{Model 1.4}(x_2)) \\
\text{Model 2.8: } \mu(x_1,x_2) &= (\text{Model 1.8}(x_1))\left(-8(x_2-1/2)^2/5+1/5\right)
\end{aligned}$$

Finally, for $d = 3$, we consider the five specifications below, which are mainly additive and interactive combinations of the above models. For Model 3.5 we set $J_n^* = 3$ in infeasible estimation.

$$\begin{aligned}
\text{Model 3.1: } \mu(x_1,x_2,x_3) &= \text{Model 1.3}(x_1) + \text{Model 1.3}(x_2) + \text{Model 1.3}(x_3) \\
\text{Model 3.2: } \mu(x_1,x_2,x_3) &= \text{Model 2.7}(x_1,x_2)\cos(8x_3) \\
\text{Model 3.3: } \mu(x_1,x_2,x_3) &= (\text{Model 1.8}(x_1))\,(\text{Model 1.8}(x_2))\,(\text{Model 1.8}(x_3)) \\
\text{Model 3.4: } \mu(x_1,x_2,x_3) &= (\text{Model 1.6}(x_1))\,(4x_2-1)(x_3-1/2) \\
\text{Model 3.5: } \mu(x_1,x_2,x_3) &= (\text{Model 1.5}(x_1))\sin(x_2)\cos(8x_3)
\end{aligned}$$

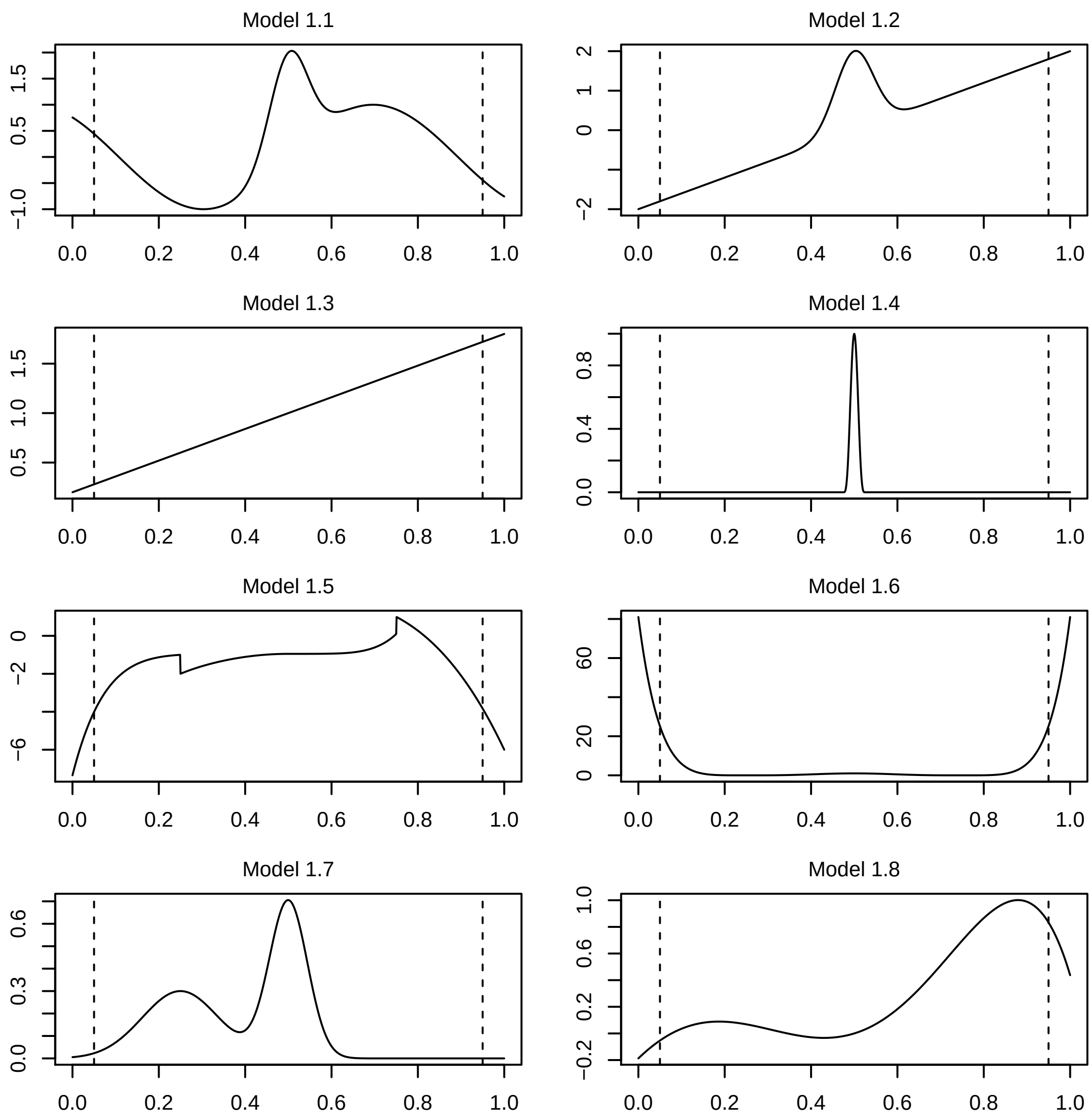

Figure C.1: Regression functions for univariate simulations. The functions are depicted over the domain [0,1], with the interval $\mathcal{X} = [0.05, 0.95]$ given in dotted lines.

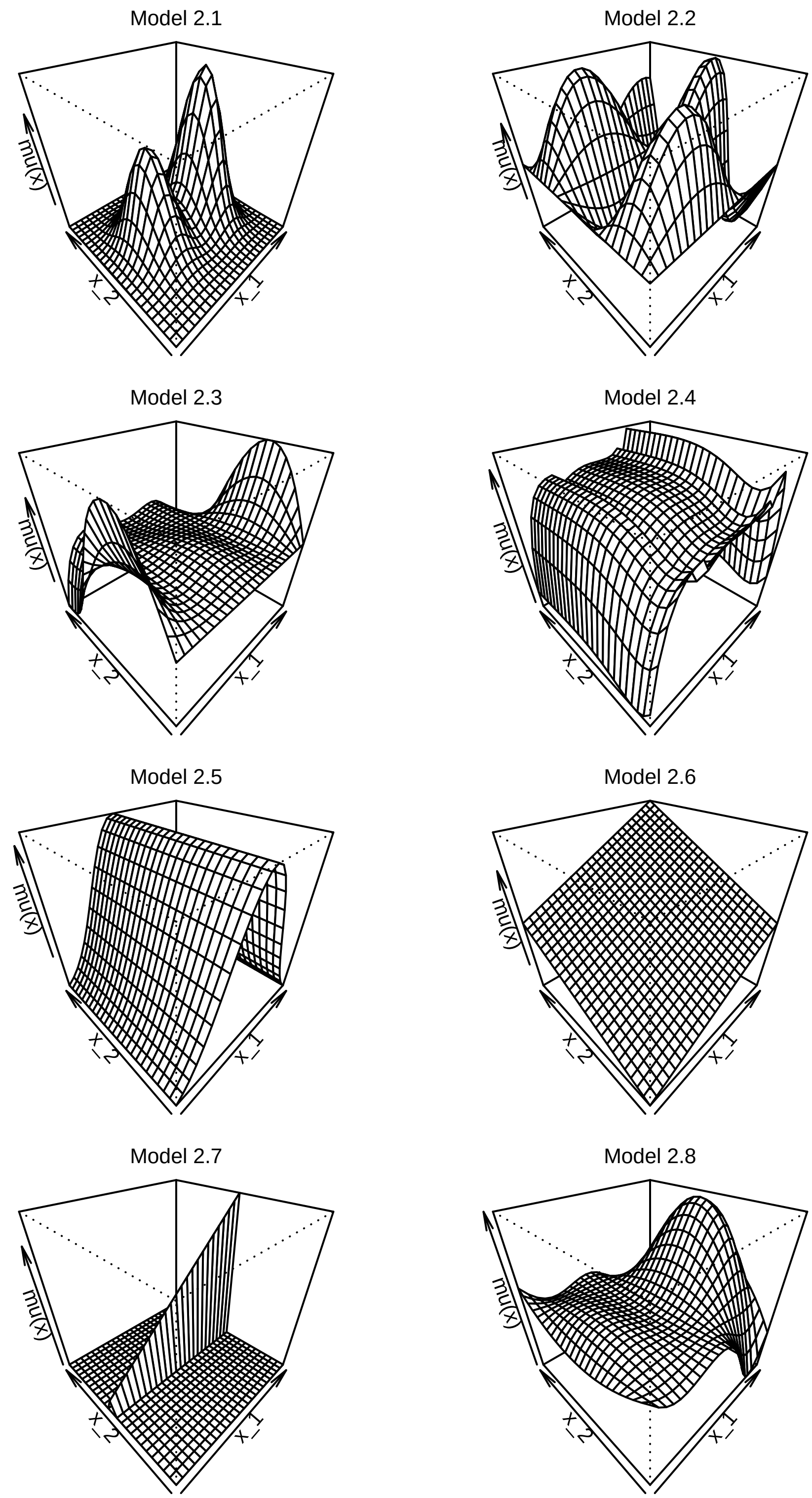

Figure C.2: Regression functions for bivariate simulations. The functions are depicted over the domain $[0,1] \times [0,1]$, although $\mathcal{X} = [0.05, 0.95] \times [0.05, 0.95]$.

C.1 Univariate Simulations

C.1.1 Uniform Cell Boundaries

Table C.1: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.305	0.219	0.124	0.169
B-splines	9	5	0.243	0.328	0.178	0.242	0.708	1.002	0.107	0.172
Partitioning	9	5	0.214	0.207	0.162	0.158	0.383	0.250	0.117	0.159
Feasible Estimation										
Local Polynomial	0.12	0.19	0.229	0.216	0.159	0.158	0.804	0.655	0.106	0.164
B-splines	4	2	0.282	0.376	0.206	0.272	0.677	1.184	0.108	0.145
Partitioning	4	2	0.257	0.199	0.184	0.146	0.571	0.413	0.106	0.112
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.307	0.219	0.123	0.169
B-splines	9	5	0.242	0.328	0.177	0.242	0.708	1.002	0.106	0.170
Partitioning	9	5	0.213	0.207	0.161	0.158	0.383	0.250	0.117	0.159
Feasible Estimation										
Local Polynomial	0.14	0.19	0.258	0.218	0.167	0.159	0.945	0.664	0.107	0.162
B-splines	3	2	0.373	0.363	0.257	0.265	1.226	1.174	0.121	0.120
Partitioning	3	2	0.328	0.192	0.206	0.142	1.032	0.384	0.104	0.118
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.071	0.045	0.072	0.081	0.102
B-splines	1	1	0.063	0.089	0.049	0.069	0.045	0.072	0.078	0.102
Partitioning	1	1	0.063	0.089	0.049	0.069	0.045	0.072	0.078	0.102
Feasible Estimation										
Local Polynomial	0.3	0.3	0.087	0.122	0.066	0.094	0.072	0.102	0.106	0.130
B-splines	2	2	0.074	0.097	0.057	0.075	0.082	0.090	0.089	0.102
Partitioning	2	2	0.085	0.119	0.065	0.090	0.117	0.248	0.092	0.103
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.142	0.166	0.090	0.119	0.929	0.866	0.108	0.156
B-splines	5	5	0.155	0.165	0.103	0.116	0.934	0.909	0.106	0.122
Partitioning	5	5	0.176	0.220	0.125	0.165	0.893	0.767	0.106	0.159
Feasible Estimation										
Local Polynomial	0.29	0.29	0.140	0.162	0.085	0.112	0.943	0.902	0.106	0.129
B-splines	2	2	0.133	0.146	0.078	0.094	0.943	0.930	0.092	0.102
Partitioning	2	2	0.140	0.152	0.084	0.106	0.939	0.789	0.095	0.105
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.447	0.224	0.332	0.164	0.098	0.120	0.323	0.159
B-splines	5	5	0.369	0.273	0.310	0.213	0.252	0.208	0.250	0.187
Partitioning	5	5	0.273	0.217	0.211	0.164	0.116	0.171	0.262	0.159
Feasible Estimation										
Local Polynomial	0.1	0.23	0.244	0.240	0.181	0.173	0.120	0.119	0.231	0.145
B-splines	5	2	0.371	0.364	0.312	0.289	0.253	0.405	0.249	0.173
Partitioning	5	2	0.275	0.290	0.212	0.223	0.119	0.521	0.261	0.134
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.280	0.207	0.222	0.161	0.297	0.201	0.279	0.185
B-splines	28	7	0.292	0.203	0.224	0.155	0.351	0.128	0.257	0.167
Partitioning	28	7	0.369	0.243	0.286	0.186	0.577	0.202	0.310	0.184
Feasible Estimation										
Local Polynomial	0.06	0.15	0.754	0.359	0.477	0.273	0.158	0.144	1.900	0.484
B-splines	8	3	1.677	1.601	1.100	1.286	0.293	0.905	3.246	2.541
Partitioning	8	3	1.441	0.769	0.814	0.592	0.304	0.140	2.795	1.182
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.08	0.12	0.135	0.159	0.106	0.125	0.216	0.179	0.109	0.164
B-splines	6	4	0.132	0.154	0.104	0.122	0.191	0.322	0.107	0.124
Partitioning	6	4	0.160	0.180	0.124	0.138	0.246	0.428	0.107	0.142
Feasible Estimation										
Local Polynomial	0.25	0.27	0.148	0.155	0.111	0.121	0.439	0.340	0.108	0.132
B-splines	2	2	0.151	0.152	0.116	0.117	0.416	0.406	0.115	0.103
Partitioning	2	2	0.155	0.140	0.116	0.107	0.419	0.318	0.114	0.114
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.12	0.33	0.112	0.121	0.087	0.093	0.109	0.093	0.107	0.125
B-splines	4	1	0.115	0.135	0.091	0.109	0.159	0.144	0.105	0.104
Partitioning	4	1	0.134	0.135	0.104	0.109	0.205	0.144	0.106	0.104
Feasible Estimation										
Local Polynomial	0.2	0.29	0.107	0.123	0.084	0.095	0.110	0.104	0.109	0.130
B-splines	2	2	0.123	0.113	0.097	0.088	0.141	0.113	0.103	0.103
Partitioning	2	2	0.126	0.131	0.097	0.102	0.127	0.261	0.104	0.105

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.2: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(1, 1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.168	0.181	0.132	0.140	0.282	0.199	0.149	0.200
B-splines	9	5	0.261	0.354	0.196	0.271	0.701	0.976	0.127	0.199
Partitioning	9	5	0.220	0.210	0.169	0.161	0.375	0.235	0.136	0.182
Feasible Estimation										
Local Polynomial	0.11	0.18	0.234	0.222	0.167	0.165	0.745	0.610	0.128	0.191
B-splines	4	3	0.325	0.410	0.243	0.308	0.765	1.182	0.131	0.140
Partitioning	4	3	0.279	0.232	0.204	0.171	0.597	0.494	0.128	0.131
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.168	0.181	0.131	0.140	0.284	0.199	0.149	0.200
B-splines	9	5	0.261	0.354	0.196	0.271	0.701	0.976	0.127	0.197
Partitioning	9	5	0.219	0.210	0.168	0.161	0.375	0.235	0.136	0.182
Feasible Estimation										
Local Polynomial	0.13	0.19	0.261	0.224	0.177	0.166	0.871	0.619	0.129	0.189
B-splines	4	3	0.342	0.403	0.255	0.305	0.920	1.175	0.143	0.131
Partitioning	4	3	0.308	0.227	0.215	0.167	0.776	0.473	0.123	0.133
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.051	0.070	0.045	0.068	0.086	0.119
B-splines	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Partitioning	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Feasible Estimation										
Local Polynomial	0.28	0.29	0.089	0.123	0.067	0.093	0.068	0.093	0.126	0.143
B-splines	2	2	0.076	0.098	0.059	0.075	0.079	0.084	0.098	0.120
Partitioning	2	2	0.088	0.120	0.067	0.091	0.114	0.223	0.102	0.121
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.153	0.174	0.094	0.121	0.927	0.864	0.128	0.172
B-splines	5	5	0.165	0.175	0.109	0.120	0.932	0.905	0.127	0.138
Partitioning	5	5	0.185	0.226	0.131	0.170	0.892	0.761	0.127	0.182
Feasible Estimation										
Local Polynomial	0.27	0.28	0.152	0.171	0.091	0.115	0.937	0.897	0.126	0.143
B-splines	2	2	0.145	0.156	0.084	0.100	0.936	0.922	0.102	0.120
Partitioning	2	2	0.152	0.160	0.092	0.111	0.927	0.757	0.106	0.122
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.426	0.229	0.307	0.168	0.092	0.107	0.267	0.176
B-splines	5	5	0.359	0.278	0.301	0.216	0.224	0.182	0.195	0.211
Partitioning	5	5	0.265	0.219	0.200	0.166	0.103	0.152	0.232	0.182
Feasible Estimation										
Local Polynomial	0.1	0.21	0.246	0.237	0.179	0.172	0.106	0.106	0.227	0.168
B-splines	5	2	0.368	0.374	0.306	0.305	0.226	0.364	0.194	0.168
Partitioning	5	2	0.271	0.291	0.204	0.228	0.113	0.475	0.231	0.142
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.268	0.203	0.210	0.155	0.250	0.177	0.320	0.214
B-splines	25	6	0.283	0.276	0.209	0.199	0.184	0.161	0.241	0.438
Partitioning	25	6	0.352	0.234	0.268	0.180	0.238	0.438	0.342	0.199
Feasible Estimation										
Local Polynomial	0.06	0.14	0.632	0.323	0.370	0.240	0.142	0.127	1.833	0.433
B-splines	8	3	1.404	1.399	0.825	1.047	0.254	0.635	3.034	2.349
Partitioning	8	3	1.209	0.697	0.616	0.510	0.266	0.127	2.692	1.273
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.08	0.12	0.139	0.162	0.108	0.125	0.200	0.162	0.131	0.192
B-splines	6	4	0.134	0.160	0.107	0.127	0.176	0.309	0.128	0.140
Partitioning	6	4	0.162	0.181	0.126	0.139	0.224	0.373	0.128	0.160
Feasible Estimation										
Local Polynomial	0.24	0.26	0.157	0.159	0.119	0.124	0.424	0.323	0.130	0.148
B-splines	2	2	0.159	0.162	0.124	0.126	0.406	0.392	0.125	0.123
Partitioning	2	2	0.164	0.141	0.125	0.109	0.396	0.285	0.126	0.129
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.13	0.33	0.110	0.120	0.085	0.091	0.095	0.084	0.128	0.136
B-splines	4	1	0.114	0.131	0.088	0.103	0.144	0.125	0.123	0.129
Partitioning	4	1	0.133	0.131	0.104	0.103	0.187	0.125	0.125	0.129
Feasible Estimation										
Local Polynomial	0.2	0.28	0.107	0.124	0.082	0.094	0.101	0.094	0.129	0.144
B-splines	3	2	0.119	0.110	0.092	0.085	0.127	0.101	0.118	0.123
Partitioning	3	2	0.122	0.130	0.094	0.100	0.114	0.234	0.118	0.123

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.3: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.173	0.182	0.133	0.136	0.252	0.176	0.231	0.307
B-splines	10	5	0.156	0.390	0.120	0.314	0.215	0.920	0.203	0.259
Partitioning	10	5	0.204	0.211	0.156	0.161	0.293	0.216	0.215	0.250
Feasible Estimation										
Local Polynomial	0.1	0.17	0.236	0.225	0.174	0.170	0.649	0.536	0.205	0.273
B-splines	5	3	0.446	0.458	0.344	0.365	1.102	1.143	0.204	0.212
Partitioning	5	3	0.327	0.271	0.230	0.207	0.749	0.547	0.196	0.195
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.173	0.182	0.132	0.136	0.253	0.176	0.230	0.307
B-splines	10	5	0.155	0.390	0.119	0.314	0.215	0.920	0.203	0.256
Partitioning	10	5	0.203	0.211	0.156	0.161	0.293	0.216	0.215	0.250
Feasible Estimation										
Local Polynomial	0.11	0.17	0.259	0.227	0.185	0.171	0.744	0.542	0.203	0.270
B-splines	4	3	0.351	0.457	0.283	0.365	0.759	1.141	0.210	0.210
Partitioning	4	3	0.301	0.270	0.231	0.205	0.550	0.542	0.187	0.195
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.068	0.045	0.063	0.098	0.173
B-splines	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Partitioning	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Feasible Estimation										
Local Polynomial	0.26	0.28	0.091	0.123	0.066	0.088	0.062	0.082	0.188	0.201
B-splines	2	2	0.078	0.098	0.059	0.073	0.075	0.078	0.126	0.182
Partitioning	2	2	0.091	0.122	0.069	0.091	0.104	0.197	0.135	0.187
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.169	0.186	0.101	0.125	0.923	0.859	0.191	0.226
B-splines	5	5	0.180	0.188	0.116	0.126	0.926	0.895	0.191	0.197
Partitioning	5	5	0.198	0.234	0.139	0.175	0.886	0.752	0.196	0.250
Feasible Estimation										
Local Polynomial	0.24	0.27	0.169	0.184	0.100	0.121	0.926	0.885	0.189	0.204
B-splines	2	2	0.163	0.172	0.096	0.108	0.923	0.910	0.140	0.185
Partitioning	2	2	0.170	0.169	0.105	0.118	0.898	0.685	0.150	0.191
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.367	0.230	0.251	0.164	0.083	0.093	0.224	0.229
B-splines	5	5	0.335	0.272	0.273	0.201	0.180	0.136	0.252	0.260
Partitioning	5	5	0.244	0.219	0.175	0.163	0.090	0.132	0.228	0.250
Feasible Estimation										
Local Polynomial	0.1	0.2	0.245	0.229	0.169	0.164	0.090	0.094	0.256	0.238
B-splines	5	2	0.399	0.365	0.307	0.297	0.188	0.278	0.296	0.202
Partitioning	5	2	0.297	0.279	0.205	0.216	0.143	0.331	0.276	0.206
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.240	0.190	0.181	0.140	0.193	0.144	0.443	0.323
B-splines	21	6	0.256	0.218	0.179	0.144	0.147	0.124	0.720	0.562
Partitioning	21	6	0.323	0.227	0.234	0.170	0.194	0.339	0.823	0.290
Feasible Estimation										
Local Polynomial	0.06	0.14	0.419	0.257	0.223	0.180	0.125	0.111	1.649	0.403
B-splines	8	3	0.934	0.986	0.445	0.621	0.200	0.279	2.501	1.768
Partitioning	8	3	0.808	0.536	0.352	0.354	0.229	0.112	2.368	1.303
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.142	0.163	0.108	0.121	0.180	0.143	0.208	0.276
B-splines	6	4	0.137	0.166	0.108	0.133	0.158	0.284	0.199	0.199
Partitioning	6	4	0.163	0.181	0.126	0.137	0.188	0.309	0.202	0.220
Feasible Estimation										
Local Polynomial	0.21	0.24	0.166	0.162	0.128	0.124	0.392	0.289	0.195	0.212
B-splines	2	2	0.175	0.174	0.138	0.140	0.399	0.365	0.151	0.196
Partitioning	2	2	0.178	0.140	0.141	0.107	0.370	0.237	0.166	0.192
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.15	0.34	0.108	0.116	0.079	0.084	0.082	0.074	0.206	0.195
B-splines	3	1	0.118	0.119	0.093	0.090	0.116	0.098	0.173	0.202
Partitioning	3	1	0.121	0.119	0.093	0.090	0.087	0.098	0.167	0.202
Feasible Estimation										
Local Polynomial	0.19	0.27	0.106	0.124	0.079	0.089	0.090	0.084	0.197	0.203
B-splines	3	2	0.109	0.106	0.083	0.079	0.109	0.085	0.159	0.187
Partitioning	3	2	0.114	0.127	0.087	0.096	0.103	0.204	0.156	0.192

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.4: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.290	0.330	0.226	0.258	0.494	0.383	0.223	0.333
B-splines	7	4	0.374	0.349	0.284	0.274	0.967	0.843	0.211	0.248
Partitioning	7	4	0.366	0.361	0.279	0.276	0.588	0.890	0.215	0.284
Feasible Estimation										
Local Polynomial	0.13	0.2	0.310	0.325	0.232	0.251	0.885	0.744	0.214	0.315
B-splines	4	2	0.368	0.415	0.275	0.313	0.922	1.185	0.224	0.229
Partitioning	4	2	0.360	0.304	0.266	0.231	0.840	0.547	0.212	0.219
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.289	0.330	0.225	0.258	0.497	0.383	0.222	0.333
B-splines	7	4	0.373	0.349	0.283	0.274	0.967	0.843	0.211	0.244
Partitioning	7	4	0.365	0.361	0.279	0.276	0.588	0.890	0.215	0.284
Feasible Estimation										
Local Polynomial	0.16	0.21	0.330	0.326	0.238	0.251	1.034	0.753	0.214	0.312
B-splines	3	2	0.422	0.402	0.304	0.304	1.294	1.183	0.218	0.213
Partitioning	3	2	0.397	0.301	0.279	0.229	1.123	0.550	0.211	0.222
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.129	0.181	0.101	0.141	0.090	0.143	0.163	0.203
B-splines	1	1	0.126	0.178	0.098	0.139	0.090	0.143	0.156	0.203
Partitioning	1	1	0.126	0.178	0.098	0.139	0.090	0.143	0.156	0.203
Feasible Estimation										
Local Polynomial	0.24	0.26	0.184	0.249	0.140	0.192	0.161	0.216	0.215	0.268
B-splines	2	2	0.160	0.197	0.124	0.153	0.170	0.187	0.187	0.204
Partitioning	2	2	0.191	0.247	0.146	0.190	0.241	0.528	0.193	0.208
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.214	0.283	0.157	0.217	0.940	0.889	0.215	0.312
B-splines	5	5	0.245	0.274	0.185	0.211	0.947	0.926	0.212	0.242
Partitioning	5	5	0.302	0.410	0.231	0.314	0.913	0.822	0.212	0.318
Feasible Estimation										
Local Polynomial	0.24	0.26	0.214	0.270	0.156	0.206	0.943	0.912	0.215	0.269
B-splines	2	2	0.195	0.225	0.140	0.167	0.953	0.941	0.189	0.204
Partitioning	2	2	0.221	0.264	0.160	0.199	0.952	0.892	0.194	0.209
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.474	0.321	0.361	0.249	0.172	0.239	0.374	0.313
B-splines	5	5	0.414	0.349	0.342	0.276	0.295	0.276	0.310	0.281
Partitioning	5	5	0.366	0.408	0.284	0.312	0.229	0.342	0.320	0.318
Feasible Estimation										
Local Polynomial	0.1	0.24	0.326	0.335	0.252	0.259	0.232	0.232	0.299	0.285
B-splines	5	2	0.475	0.417	0.386	0.331	0.322	0.437	0.299	0.246
Partitioning	5	2	0.400	0.383	0.309	0.295	0.308	0.688	0.312	0.227
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.486	0.377	0.385	0.295	0.507	0.372	0.484	0.347
B-splines	21	6	0.512	0.386	0.393	0.298	0.372	0.310	0.640	0.435
Partitioning	21	6	0.640	0.448	0.496	0.343	0.480	1.011	0.628	0.338
Feasible Estimation										
Local Polynomial	0.06	0.15	0.810	0.451	0.559	0.354	0.305	0.283	1.955	0.629
B-splines	8	3	1.695	1.613	1.136	1.292	0.439	0.917	3.252	2.547
Partitioning	8	3	1.475	0.814	0.914	0.641	0.585	0.267	2.804	1.199
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.11	0.14	0.237	0.299	0.185	0.233	0.332	0.315	0.214	0.316
B-splines	4	3	0.222	0.249	0.175	0.196	0.373	0.449	0.210	0.206
Partitioning	4	3	0.265	0.316	0.205	0.242	0.443	0.330	0.211	0.237
Feasible Estimation										
Local Polynomial	0.23	0.26	0.218	0.266	0.169	0.208	0.445	0.379	0.216	0.271
B-splines	2	2	0.208	0.229	0.162	0.180	0.449	0.436	0.197	0.204
Partitioning	2	2	0.230	0.257	0.176	0.198	0.460	0.549	0.202	0.214
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.16	0.38	0.202	0.222	0.157	0.171	0.187	0.170	0.218	0.221
B-splines	3	1	0.202	0.205	0.160	0.161	0.192	0.190	0.203	0.204
Partitioning	3	1	0.229	0.205	0.178	0.161	0.183	0.190	0.205	0.204
Feasible Estimation										
Local Polynomial	0.21	0.26	0.197	0.250	0.153	0.193	0.188	0.217	0.215	0.268
B-splines	2	2	0.193	0.201	0.151	0.157	0.221	0.195	0.199	0.204
Partitioning	2	2	0.217	0.251	0.168	0.193	0.251	0.532	0.203	0.209

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.5: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.295	0.335	0.230	0.259	0.458	0.347	0.268	0.390
B-splines	7	4	0.397	0.367	0.308	0.292	0.955	0.813	0.255	0.281
Partitioning	7	4	0.374	0.363	0.289	0.279	0.573	0.780	0.257	0.319
Feasible Estimation										
Local Polynomial	0.12	0.2	0.321	0.334	0.243	0.257	0.844	0.705	0.257	0.356
B-splines	4	2	0.376	0.443	0.291	0.342	0.814	1.156	0.265	0.251
Partitioning	4	2	0.368	0.318	0.281	0.242	0.734	0.511	0.251	0.257
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.294	0.335	0.229	0.259	0.460	0.347	0.268	0.390
B-splines	7	4	0.396	0.366	0.307	0.291	0.954	0.813	0.255	0.278
Partitioning	7	4	0.374	0.363	0.288	0.279	0.573	0.780	0.257	0.319
Feasible Estimation										
Local Polynomial	0.15	0.2	0.343	0.335	0.252	0.258	0.977	0.712	0.257	0.353
B-splines	3	2	0.435	0.433	0.327	0.335	1.166	1.153	0.263	0.246
Partitioning	3	2	0.411	0.315	0.300	0.240	1.024	0.509	0.247	0.258
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.130	0.183	0.101	0.140	0.090	0.136	0.171	0.238
B-splines	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Partitioning	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Feasible Estimation										
Local Polynomial	0.23	0.26	0.188	0.250	0.141	0.189	0.148	0.194	0.256	0.296
B-splines	2	2	0.163	0.199	0.126	0.153	0.165	0.174	0.209	0.241
Partitioning	2	2	0.194	0.249	0.149	0.190	0.227	0.473	0.218	0.243
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.222	0.288	0.160	0.217	0.937	0.884	0.256	0.344
B-splines	5	5	0.252	0.281	0.190	0.212	0.944	0.921	0.253	0.274
Partitioning	5	5	0.307	0.414	0.236	0.319	0.909	0.804	0.255	0.364
Feasible Estimation										
Local Polynomial	0.23	0.26	0.224	0.277	0.160	0.206	0.937	0.907	0.256	0.298
B-splines	2	2	0.205	0.233	0.146	0.171	0.946	0.934	0.211	0.241
Partitioning	2	2	0.229	0.269	0.166	0.202	0.939	0.839	0.219	0.244
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.455	0.324	0.337	0.249	0.158	0.214	0.347	0.347
B-splines	5	5	0.407	0.355	0.333	0.277	0.265	0.246	0.293	0.318
Partitioning	5	5	0.361	0.411	0.278	0.315	0.202	0.303	0.320	0.364
Feasible Estimation										
Local Polynomial	0.11	0.23	0.329	0.334	0.251	0.256	0.203	0.210	0.319	0.321
B-splines	5	2	0.486	0.417	0.388	0.336	0.298	0.394	0.287	0.269
Partitioning	5	2	0.404	0.374	0.308	0.292	0.301	0.610	0.308	0.259
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.09	0.463	0.370	0.363	0.285	0.433	0.327	0.550	0.407
B-splines	19	5	0.495	0.598	0.368	0.466	0.316	0.466	1.113	1.088
Partitioning	19	5	0.612	0.430	0.468	0.330	0.413	0.303	1.129	0.392
Feasible Estimation										
Local Polynomial	0.06	0.15	0.700	0.424	0.459	0.324	0.271	0.249	1.916	0.628
B-splines	8	3	1.438	1.412	0.877	1.055	0.390	0.650	3.053	2.359
Partitioning	8	3	1.259	0.748	0.727	0.564	0.502	0.240	2.720	1.295
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.1	0.13	0.244	0.303	0.188	0.233	0.311	0.284	0.257	0.365
B-splines	5	4	0.256	0.260	0.203	0.204	0.442	0.360	0.254	0.258
Partitioning	5	4	0.297	0.359	0.231	0.276	0.344	0.715	0.255	0.319
Feasible Estimation										
Local Polynomial	0.22	0.25	0.226	0.270	0.175	0.208	0.432	0.363	0.259	0.301
B-splines	2	2	0.217	0.237	0.170	0.186	0.440	0.421	0.222	0.242
Partitioning	2	2	0.238	0.259	0.184	0.200	0.442	0.493	0.229	0.247
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.17	0.39	0.200	0.221	0.153	0.167	0.165	0.155	0.260	0.251
B-splines	3	1	0.201	0.204	0.157	0.157	0.175	0.171	0.232	0.242
Partitioning	3	1	0.229	0.204	0.178	0.157	0.166	0.171	0.236	0.242
Feasible Estimation										
Local Polynomial	0.21	0.26	0.199	0.251	0.151	0.189	0.170	0.195	0.257	0.297
B-splines	2	2	0.192	0.203	0.149	0.156	0.202	0.179	0.225	0.242
Partitioning	2	2	0.216	0.251	0.167	0.193	0.231	0.477	0.232	0.244

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.6: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.303	0.338	0.230	0.251	0.410	0.307	0.422	0.573
B-splines	7	5	0.428	0.447	0.342	0.360	0.919	0.931	0.406	0.425
Partitioning	7	5	0.383	0.406	0.296	0.308	0.546	0.314	0.409	0.499
Feasible Estimation										
Local Polynomial	0.12	0.19	0.336	0.342	0.255	0.260	0.781	0.640	0.406	0.493
B-splines	4	3	0.405	0.481	0.324	0.384	0.765	1.098	0.391	0.407
Partitioning	4	3	0.384	0.339	0.301	0.259	0.623	0.483	0.377	0.387
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.07	0.1	0.302	0.338	0.230	0.251	0.412	0.307	0.422	0.573
B-splines	7	5	0.427	0.447	0.341	0.360	0.919	0.931	0.406	0.424
Partitioning	7	5	0.382	0.406	0.295	0.308	0.546	0.314	0.409	0.499
Feasible Estimation										
Local Polynomial	0.13	0.19	0.356	0.342	0.266	0.261	0.883	0.644	0.401	0.491
B-splines	4	3	0.443	0.475	0.350	0.380	0.974	1.095	0.391	0.393
Partitioning	4	3	0.422	0.337	0.322	0.257	0.850	0.479	0.353	0.387
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.129	0.182	0.100	0.136	0.090	0.125	0.197	0.346
B-splines	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Partitioning	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Feasible Estimation										
Local Polynomial	0.22	0.26	0.190	0.250	0.137	0.179	0.133	0.170	0.388	0.411
B-splines	2	2	0.166	0.199	0.126	0.148	0.150	0.158	0.274	0.366
Partitioning	2	2	0.197	0.249	0.150	0.187	0.201	0.407	0.294	0.377
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.234	0.294	0.162	0.212	0.930	0.874	0.382	0.453
B-splines	5	5	0.262	0.289	0.193	0.212	0.933	0.907	0.381	0.392
Partitioning	5	5	0.315	0.418	0.241	0.319	0.899	0.785	0.392	0.499
Feasible Estimation										
Local Polynomial	0.22	0.25	0.238	0.285	0.164	0.204	0.927	0.893	0.389	0.414
B-splines	2	2	0.220	0.244	0.155	0.175	0.930	0.918	0.282	0.368
Partitioning	2	2	0.244	0.275	0.176	0.204	0.910	0.757	0.301	0.380
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.402	0.323	0.283	0.238	0.139	0.185	0.401	0.454
B-splines	5	5	0.386	0.349	0.305	0.261	0.217	0.199	0.416	0.427
Partitioning	5	5	0.346	0.410	0.260	0.309	0.177	0.263	0.411	0.499
Feasible Estimation										
Local Polynomial	0.12	0.21	0.328	0.328	0.239	0.242	0.170	0.184	0.436	0.451
B-splines	4	2	0.495	0.411	0.371	0.328	0.263	0.314	0.450	0.384
Partitioning	4	2	0.417	0.366	0.300	0.284	0.298	0.480	0.448	0.396
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.03	0.1	0.418	0.350	0.314	0.259	0.333	0.266	0.756	0.594
B-splines	16	5	0.449	0.483	0.314	0.346	0.385	0.341	0.723	1.186
Partitioning	16	5	0.555	0.416	0.410	0.312	0.588	0.263	1.129	0.557
Feasible Estimation										
Local Polynomial	0.06	0.15	0.515	0.371	0.318	0.268	0.231	0.215	1.810	0.703
B-splines	7	3	1.105	1.005	0.581	0.635	0.302	0.307	2.442	1.793
Partitioning	7	3	0.979	0.599	0.504	0.418	0.295	0.210	2.430	1.342
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.1	0.13	0.250	0.305	0.187	0.225	0.282	0.252	0.409	0.511
B-splines	5	4	0.266	0.264	0.211	0.202	0.422	0.326	0.382	0.385
Partitioning	5	4	0.301	0.360	0.234	0.272	0.322	0.591	0.392	0.439
Feasible Estimation										
Local Polynomial	0.21	0.24	0.236	0.273	0.181	0.204	0.407	0.333	0.391	0.420
B-splines	2	2	0.230	0.246	0.181	0.193	0.419	0.393	0.291	0.374
Partitioning	2	2	0.251	0.262	0.196	0.198	0.406	0.423	0.313	0.381
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.19	0.4	0.193	0.215	0.142	0.156	0.142	0.139	0.386	0.386
B-splines	2	1	0.162	0.196	0.124	0.146	0.169	0.146	0.254	0.357
Partitioning	2	1	0.185	0.196	0.143	0.146	0.236	0.146	0.268	0.357
Feasible Estimation										
Local Polynomial	0.2	0.25	0.200	0.250	0.146	0.180	0.152	0.171	0.392	0.412
B-splines	3	2	0.186	0.202	0.142	0.151	0.176	0.161	0.299	0.367
Partitioning	3	2	0.213	0.252	0.162	0.190	0.204	0.411	0.312	0.379

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.7: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.125	0.132	0.097	0.103	0.240	0.168	0.093	0.118
B-splines	10	5	0.109	0.317	0.086	0.230	0.242	1.000	0.077	0.149
Partitioning	10	5	0.144	0.152	0.111	0.116	0.296	0.222	0.089	0.111
Feasible Estimation										
Local Polynomial	0.1	0.18	0.187	0.178	0.126	0.126	0.686	0.576	0.075	0.127
B-splines	5	3	0.351	0.375	0.236	0.268	1.122	1.244	0.074	0.117
Partitioning	5	3	0.255	0.210	0.159	0.144	0.766	0.571	0.075	0.083
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.124	0.132	0.097	0.103	0.242	0.168	0.093	0.118
B-splines	10	5	0.109	0.317	0.085	0.230	0.242	1.000	0.077	0.146
Partitioning	10	5	0.144	0.152	0.111	0.116	0.296	0.222	0.089	0.111
Feasible Estimation										
Local Polynomial	0.12	0.18	0.215	0.180	0.134	0.127	0.822	0.585	0.075	0.125
B-splines	4	3	0.269	0.373	0.196	0.267	0.734	1.241	0.079	0.104
Partitioning	4	3	0.241	0.208	0.164	0.142	0.599	0.563	0.074	0.083
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.065	0.036	0.050	0.032	0.051	0.057	0.072
B-splines	1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Partitioning	1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Feasible Estimation										
Local Polynomial	0.3	0.3	0.062	0.087	0.047	0.067	0.051	0.072	0.075	0.090
B-splines	2	2	0.053	0.069	0.041	0.054	0.058	0.065	0.063	0.071
Partitioning	2	2	0.061	0.084	0.047	0.064	0.082	0.177	0.065	0.073
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.127	0.138	0.071	0.091	0.927	0.864	0.076	0.109
B-splines	5	5	0.135	0.140	0.081	0.090	0.932	0.905	0.075	0.087
Partitioning	5	5	0.146	0.170	0.096	0.123	0.891	0.760	0.075	0.111
Feasible Estimation										
Local Polynomial	0.28	0.28	0.126	0.137	0.069	0.087	0.939	0.898	0.075	0.092
B-splines	2	2	0.123	0.129	0.064	0.075	0.940	0.925	0.066	0.072
Partitioning	2	2	0.127	0.128	0.069	0.083	0.934	0.763	0.068	0.074
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.443	0.205	0.327	0.143	0.079	0.084	0.315	0.114
B-splines	5	5	0.360	0.259	0.305	0.201	0.243	0.194	0.238	0.165
Partitioning	5	5	0.255	0.165	0.196	0.123	0.082	0.121	0.251	0.111
Feasible Estimation										
Local Polynomial	0.09	0.21	0.207	0.210	0.150	0.146	0.091	0.084	0.202	0.112
B-splines	6	2	0.288	0.353	0.222	0.281	0.168	0.396	0.256	0.158
Partitioning	6	2	0.235	0.271	0.179	0.209	0.153	0.468	0.248	0.114
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.211	0.144	0.168	0.113	0.223	0.149	0.205	0.131
B-splines	32	7	0.221	0.177	0.170	0.134	0.268	0.095	0.168	0.125
Partitioning	32	7	0.280	0.176	0.217	0.136	0.424	0.143	0.221	0.138
Feasible Estimation										
Local Polynomial	0.05	0.13	0.618	0.228	0.387	0.172	0.121	0.106	1.469	0.120
B-splines	9	4	1.405	1.105	0.873	0.885	0.101	0.819	2.871	1.799
Partitioning	9	4	1.212	0.484	0.659	0.345	0.120	0.264	2.334	0.670
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.103	0.117	0.080	0.092	0.173	0.137	0.079	0.116
B-splines	7	4	0.135	0.130	0.103	0.102	0.340	0.311	0.076	0.096
Partitioning	7	4	0.129	0.128	0.099	0.098	0.204	0.317	0.076	0.100
Feasible Estimation										
Local Polynomial	0.23	0.25	0.133	0.126	0.097	0.097	0.424	0.318	0.077	0.095
B-splines	2	2	0.139	0.136	0.106	0.102	0.405	0.396	0.084	0.073
Partitioning	2	2	0.140	0.108	0.104	0.083	0.396	0.238	0.082	0.087
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.11	0.3	0.084	0.091	0.066	0.070	0.082	0.069	0.076	0.097
B-splines	4	2	0.092	0.076	0.072	0.059	0.130	0.072	0.074	0.072
Partitioning	4	2	0.101	0.090	0.079	0.070	0.153	0.203	0.075	0.073
Feasible Estimation										
Local Polynomial	0.17	0.29	0.082	0.088	0.064	0.068	0.082	0.074	0.078	0.090
B-splines	3	2	0.110	0.089	0.090	0.070	0.139	0.092	0.074	0.073
Partitioning	3	2	0.101	0.100	0.079	0.078	0.092	0.194	0.073	0.074

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.8: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(1, 1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.220	0.151	0.111	0.139
B-splines	11	6	0.185	0.207	0.138	0.165	0.514	0.495	0.092	0.137
Partitioning	11	6	0.166	0.157	0.128	0.121	0.273	0.349	0.112	0.136
Feasible Estimation										
Local Polynomial	0.1	0.17	0.190	0.181	0.132	0.131	0.631	0.536	0.090	0.150
B-splines	5	3	0.407	0.408	0.287	0.305	1.218	1.216	0.092	0.111
Partitioning	5	3	0.283	0.230	0.176	0.162	0.820	0.581	0.089	0.093
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.221	0.151	0.111	0.139
B-splines	10	6	0.109	0.207	0.085	0.165	0.227	0.495	0.089	0.138
Partitioning	10	6	0.144	0.157	0.112	0.121	0.273	0.349	0.102	0.136
Feasible Estimation										
Local Polynomial	0.11	0.17	0.216	0.183	0.141	0.132	0.750	0.544	0.089	0.148
B-splines	4	3	0.305	0.408	0.230	0.305	0.773	1.216	0.096	0.102
Partitioning	4	3	0.257	0.229	0.181	0.161	0.578	0.580	0.088	0.093
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.049	0.031	0.047	0.061	0.084
B-splines	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Partitioning	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Feasible Estimation										
Local Polynomial	0.28	0.29	0.062	0.087	0.047	0.065	0.047	0.065	0.088	0.099
B-splines	2	2	0.053	0.069	0.041	0.053	0.055	0.059	0.070	0.084
Partitioning	2	2	0.062	0.085	0.047	0.065	0.079	0.161	0.073	0.084
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.137	0.147	0.075	0.094	0.927	0.863	0.089	0.119
B-splines	5	5	0.145	0.149	0.085	0.094	0.931	0.904	0.088	0.097
Partitioning	5	5	0.155	0.175	0.101	0.128	0.892	0.758	0.089	0.127
Feasible Estimation										
Local Polynomial	0.26	0.28	0.137	0.146	0.073	0.090	0.935	0.894	0.088	0.101
B-splines	2	2	0.134	0.140	0.069	0.080	0.935	0.921	0.076	0.085
Partitioning	2	2	0.138	0.135	0.075	0.088	0.925	0.741	0.079	0.086
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.421	0.210	0.301	0.147	0.075	0.076	0.251	0.125
B-splines	5	5	0.351	0.264	0.296	0.205	0.217	0.170	0.169	0.189
Partitioning	5	5	0.247	0.167	0.185	0.124	0.075	0.106	0.212	0.127
Feasible Estimation										
Local Polynomial	0.09	0.2	0.209	0.209	0.147	0.146	0.080	0.077	0.195	0.129
B-splines	6	2	0.291	0.356	0.225	0.290	0.158	0.343	0.208	0.143
Partitioning	6	2	0.232	0.266	0.172	0.207	0.129	0.379	0.221	0.115
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.201	0.141	0.158	0.110	0.184	0.128	0.237	0.151
B-splines	29	7	0.212	0.160	0.158	0.116	0.137	0.092	0.238	0.156
Partitioning	29	7	0.267	0.174	0.204	0.133	0.175	0.125	0.269	0.152
Feasible Estimation										
Local Polynomial	0.05	0.13	0.517	0.209	0.298	0.155	0.107	0.094	1.430	0.152
B-splines	9	4	1.169	0.939	0.642	0.723	0.099	0.635	2.775	1.738
Partitioning	9	4	1.011	0.411	0.492	0.280	0.109	0.237	2.324	0.691
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.104	0.118	0.081	0.091	0.160	0.123	0.093	0.135
B-splines	7	4	0.142	0.136	0.111	0.108	0.336	0.300	0.090	0.109
Partitioning	7	4	0.131	0.128	0.102	0.098	0.201	0.281	0.090	0.111
Feasible Estimation										
Local Polynomial	0.21	0.24	0.140	0.128	0.103	0.099	0.408	0.300	0.091	0.106
B-splines	2	2	0.149	0.145	0.114	0.112	0.407	0.383	0.092	0.088
Partitioning	2	2	0.149	0.107	0.113	0.083	0.387	0.207	0.092	0.097
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.11	0.31	0.082	0.089	0.063	0.067	0.071	0.061	0.090	0.102
B-splines	4	2	0.089	0.075	0.069	0.057	0.118	0.066	0.086	0.085
Partitioning	4	2	0.098	0.090	0.076	0.069	0.139	0.182	0.087	0.085
Feasible Estimation										
Local Polynomial	0.17	0.28	0.080	0.087	0.062	0.066	0.075	0.067	0.091	0.100
B-splines	3	2	0.105	0.085	0.084	0.065	0.124	0.078	0.088	0.088
Partitioning	3	2	0.097	0.097	0.075	0.075	0.085	0.173	0.083	0.088

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.9: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 1.1					
Infeasible Estimation										
Local Polynomial	0.04	0.08	0.132	0.136	0.102	0.101	0.196	0.134	0.169	0.209
B-splines	11	6	0.203	0.225	0.157	0.186	0.498	0.463	0.137	0.195
Partitioning	11	6	0.173	0.159	0.133	0.121	0.264	0.282	0.158	0.194
Feasible Estimation										
Local Polynomial	0.09	0.16	0.191	0.185	0.139	0.138	0.545	0.470	0.140	0.218
B-splines	5	3	0.381	0.455	0.275	0.363	0.951	1.150	0.133	0.141
Partitioning	5	3	0.274	0.256	0.182	0.190	0.653	0.566	0.138	0.134
					Model 1.2					
Infeasible Estimation										
Local Polynomial	0.04	0.08	0.131	0.136	0.101	0.101	0.197	0.134	0.169	0.209
B-splines	11	6	0.203	0.225	0.157	0.186	0.498	0.463	0.137	0.196
Partitioning	11	6	0.172	0.159	0.133	0.121	0.264	0.282	0.158	0.194
Feasible Estimation										
Local Polynomial	0.1	0.16	0.212	0.187	0.149	0.139	0.633	0.476	0.139	0.216
B-splines	5	3	0.446	0.454	0.343	0.363	1.145	1.150	0.147	0.142
Partitioning	5	3	0.314	0.256	0.203	0.190	0.783	0.566	0.133	0.134
					Model 1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.048	0.032	0.044	0.070	0.120
B-splines	1	1	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Partitioning	1	1	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Feasible Estimation										
Local Polynomial	0.26	0.28	0.064	0.087	0.046	0.063	0.044	0.057	0.129	0.137
B-splines	2	2	0.055	0.069	0.042	0.052	0.053	0.054	0.088	0.126
Partitioning	2	2	0.065	0.086	0.049	0.065	0.075	0.142	0.094	0.128
					Model 1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.156	0.162	0.084	0.101	0.923	0.858	0.131	0.156
B-splines	5	5	0.163	0.166	0.095	0.103	0.926	0.895	0.132	0.135
Partitioning	5	5	0.171	0.187	0.111	0.136	0.887	0.753	0.134	0.173
Feasible Estimation										
Local Polynomial	0.23	0.26	0.156	0.162	0.084	0.099	0.923	0.883	0.131	0.140
B-splines	2	2	0.154	0.157	0.083	0.091	0.921	0.907	0.104	0.128
Partitioning	2	2	0.157	0.146	0.089	0.097	0.903	0.681	0.110	0.130
					Model 1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.361	0.211	0.244	0.146	0.069	0.065	0.162	0.163
B-splines	5	5	0.323	0.259	0.266	0.192	0.173	0.126	0.174	0.222
Partitioning	5	5	0.228	0.168	0.160	0.124	0.064	0.092	0.168	0.173
Feasible Estimation										
Local Polynomial	0.09	0.18	0.210	0.203	0.140	0.140	0.067	0.068	0.203	0.183
B-splines	5	3	0.317	0.317	0.259	0.256	0.166	0.234	0.181	0.143
Partitioning	5	3	0.227	0.227	0.159	0.165	0.070	0.145	0.174	0.145
					Model 1.6					
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.184	0.141	0.138	0.104	0.142	0.104	0.329	0.220
B-splines	24	6	0.194	0.196	0.135	0.123	0.166	0.095	0.301	0.526
Partitioning	24	6	0.243	0.166	0.178	0.124	0.254	0.251	0.370	0.202
Feasible Estimation										
Local Polynomial	0.05	0.13	0.343	0.169	0.178	0.120	0.094	0.081	1.306	0.244
B-splines	9	4	0.767	0.725	0.331	0.492	0.095	0.387	2.441	1.599
Partitioning	9	4	0.671	0.342	0.274	0.212	0.097	0.189	2.175	0.847
					Model 1.7					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.107	0.120	0.082	0.090	0.144	0.109	0.144	0.195
B-splines	7	5	0.154	0.161	0.124	0.129	0.325	0.334	0.139	0.144
Partitioning	7	5	0.136	0.144	0.106	0.110	0.192	0.112	0.140	0.173
Feasible Estimation										
Local Polynomial	0.18	0.22	0.147	0.130	0.112	0.100	0.373	0.264	0.135	0.151
B-splines	3	2	0.169	0.159	0.131	0.128	0.412	0.361	0.114	0.143
Partitioning	3	2	0.166	0.108	0.130	0.082	0.380	0.164	0.127	0.132
					Model 1.8					
Infeasible Estimation										
Local Polynomial	0.13	0.32	0.080	0.087	0.059	0.062	0.061	0.054	0.137	0.136
B-splines	4	2	0.083	0.074	0.063	0.055	0.100	0.058	0.125	0.128
Partitioning	4	2	0.096	0.090	0.073	0.068	0.118	0.154	0.128	0.130
Feasible Estimation										
Local Polynomial	0.16	0.27	0.079	0.088	0.059	0.064	0.068	0.059	0.137	0.138
B-splines	3	2	0.096	0.079	0.075	0.058	0.108	0.063	0.129	0.133
Partitioning	3	2	0.091	0.093	0.070	0.070	0.084	0.149	0.117	0.135

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.10: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.218	0.243	0.170	0.190	0.394	0.293	0.164	0.234
B-splines	8	5	0.198	0.353	0.156	0.267	0.271	1.008	0.149	0.210
Partitioning	8	5	0.257	0.288	0.199	0.221	0.426	0.306	0.156	0.222
Feasible Estimation										
Local Polynomial	0.11	0.19	0.247	0.254	0.181	0.193	0.761	0.645	0.150	0.241
B-splines	4	3	0.327	0.395	0.240	0.291	0.852	1.218	0.149	0.174
Partitioning	4	3	0.294	0.262	0.215	0.195	0.670	0.518	0.150	0.159
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.217	0.243	0.170	0.190	0.396	0.293	0.163	0.234
B-splines	8	5	0.197	0.353	0.155	0.267	0.271	1.008	0.149	0.209
Partitioning	8	5	0.257	0.288	0.198	0.221	0.426	0.306	0.156	0.222
Feasible Estimation										
Local Polynomial	0.13	0.19	0.271	0.255	0.189	0.194	0.907	0.653	0.151	0.239
B-splines	4	3	0.347	0.387	0.252	0.286	1.017	1.214	0.155	0.160
Partitioning	4	3	0.322	0.259	0.224	0.193	0.881	0.505	0.148	0.160
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.093	0.129	0.072	0.101	0.064	0.102	0.115	0.143
B-splines	1	1	0.090	0.127	0.070	0.099	0.063	0.102	0.109	0.143
Partitioning	1	1	0.090	0.127	0.070	0.099	0.063	0.102	0.109	0.143
Feasible Estimation										
Local Polynomial	0.24	0.27	0.132	0.177	0.101	0.137	0.114	0.151	0.151	0.188
B-splines	2	2	0.114	0.140	0.089	0.110	0.123	0.134	0.132	0.143
Partitioning	2	2	0.136	0.176	0.104	0.135	0.170	0.381	0.136	0.146
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.171	0.214	0.118	0.160	0.933	0.877	0.152	0.218
B-splines	5	5	0.191	0.209	0.138	0.155	0.939	0.915	0.150	0.173
Partitioning	5	5	0.227	0.298	0.169	0.227	0.902	0.790	0.150	0.222
Feasible Estimation										
Local Polynomial	0.24	0.26	0.171	0.206	0.118	0.152	0.935	0.900	0.151	0.190
B-splines	2	2	0.159	0.178	0.106	0.126	0.945	0.931	0.133	0.143
Partitioning	2	2	0.176	0.199	0.121	0.146	0.937	0.815	0.137	0.147
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.457	0.261	0.344	0.199	0.128	0.168	0.342	0.221
B-splines	5	5	0.385	0.302	0.321	0.237	0.267	0.233	0.272	0.222
Partitioning	5	5	0.308	0.295	0.239	0.226	0.160	0.242	0.283	0.222
Feasible Estimation										
Local Polynomial	0.09	0.22	0.264	0.272	0.201	0.206	0.176	0.168	0.246	0.212
B-splines	5	2	0.367	0.376	0.302	0.299	0.238	0.410	0.278	0.201
Partitioning	5	2	0.306	0.318	0.237	0.246	0.211	0.543	0.283	0.175
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.368	0.279	0.293	0.218	0.385	0.276	0.361	0.247
B-splines	24	6	0.388	0.336	0.297	0.259	0.463	0.233	0.250	0.386
Partitioning	24	6	0.486	0.324	0.378	0.249	0.726	0.708	0.373	0.239
Feasible Estimation										
Local Polynomial	0.05	0.14	0.659	0.294	0.450	0.233	0.233	0.208	1.521	0.229
B-splines	9	3	1.415	1.391	0.904	1.101	0.178	0.898	2.873	2.210
Partitioning	9	3	1.234	0.675	0.733	0.508	0.223	0.394	2.337	0.990
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.13	0.180	0.218	0.141	0.171	0.271	0.239	0.152	0.226
B-splines	5	4	0.196	0.195	0.153	0.154	0.434	0.343	0.150	0.164
Partitioning	5	4	0.216	0.254	0.166	0.195	0.323	0.596	0.150	0.199
Feasible Estimation										
Local Polynomial	0.22	0.25	0.175	0.199	0.135	0.156	0.424	0.345	0.152	0.193
B-splines	2	2	0.174	0.183	0.134	0.143	0.430	0.415	0.143	0.144
Partitioning	2	2	0.186	0.189	0.142	0.146	0.423	0.404	0.145	0.154
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.14	0.36	0.152	0.164	0.119	0.127	0.142	0.126	0.154	0.164
B-splines	3	1	0.158	0.163	0.127	0.129	0.166	0.161	0.143	0.144
Partitioning	3	1	0.169	0.163	0.132	0.129	0.135	0.161	0.145	0.144
Feasible Estimation										
Local Polynomial	0.19	0.26	0.146	0.177	0.114	0.137	0.141	0.152	0.152	0.188
B-splines	3	2	0.154	0.147	0.121	0.115	0.188	0.144	0.144	0.143
Partitioning	3	2	0.165	0.180	0.128	0.139	0.188	0.384	0.145	0.147

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.11: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(1, 1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.222	0.244	0.173	0.189	0.363	0.263	0.194	0.274
B-splines	8	5	0.199	0.377	0.157	0.293	0.240	0.984	0.177	0.242
Partitioning	8	5	0.257	0.289	0.200	0.222	0.377	0.282	0.181	0.253
Feasible Estimation										
Local Polynomial	0.11	0.18	0.255	0.260	0.189	0.198	0.723	0.612	0.179	0.278
B-splines	4	3	0.379	0.425	0.283	0.323	0.981	1.197	0.177	0.182
Partitioning	4	3	0.318	0.280	0.233	0.212	0.714	0.533	0.178	0.182
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.221	0.244	0.172	0.189	0.365	0.263	0.193	0.274
B-splines	8	5	0.198	0.377	0.156	0.293	0.240	0.984	0.177	0.240
Partitioning	8	5	0.257	0.289	0.200	0.222	0.376	0.282	0.181	0.253
Feasible Estimation										
Local Polynomial	0.12	0.18	0.277	0.261	0.198	0.199	0.847	0.618	0.179	0.275
B-splines	4	3	0.351	0.420	0.267	0.320	0.884	1.194	0.185	0.176
Partitioning	4	3	0.325	0.277	0.238	0.210	0.753	0.525	0.173	0.183
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.091	0.128	0.071	0.099	0.062	0.094	0.121	0.167
B-splines	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Partitioning	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Feasible Estimation										
Local Polynomial	0.24	0.26	0.131	0.176	0.099	0.133	0.103	0.136	0.178	0.206
B-splines	2	2	0.115	0.140	0.089	0.107	0.115	0.123	0.147	0.169
Partitioning	2	2	0.137	0.176	0.105	0.134	0.159	0.340	0.154	0.170
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.178	0.218	0.120	0.159	0.933	0.875	0.179	0.239
B-splines	5	5	0.197	0.215	0.141	0.157	0.938	0.912	0.177	0.192
Partitioning	5	5	0.232	0.301	0.173	0.230	0.902	0.782	0.177	0.253
Feasible Estimation										
Local Polynomial	0.23	0.26	0.179	0.212	0.120	0.153	0.933	0.898	0.179	0.209
B-splines	2	2	0.169	0.185	0.110	0.128	0.942	0.926	0.151	0.169
Partitioning	2	2	0.185	0.204	0.126	0.149	0.932	0.784	0.158	0.170
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.435	0.264	0.317	0.200	0.118	0.151	0.293	0.242
B-splines	5	5	0.375	0.306	0.312	0.239	0.239	0.207	0.227	0.252
Partitioning	5	5	0.302	0.296	0.230	0.227	0.145	0.211	0.260	0.253
Feasible Estimation										
Local Polynomial	0.09	0.21	0.266	0.270	0.199	0.203	0.153	0.151	0.252	0.243
B-splines	5	2	0.368	0.379	0.303	0.307	0.224	0.364	0.236	0.204
Partitioning	5	2	0.301	0.314	0.230	0.246	0.168	0.458	0.263	0.191
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.351	0.271	0.276	0.209	0.317	0.235	0.415	0.286
B-splines	22	6	0.371	0.306	0.276	0.225	0.391	0.200	0.400	0.459
Partitioning	22	6	0.465	0.320	0.356	0.246	0.607	0.626	0.448	0.274
Feasible Estimation										
Local Polynomial	0.05	0.14	0.568	0.280	0.367	0.217	0.203	0.183	1.509	0.270
B-splines	9	3	1.181	1.244	0.678	0.929	0.166	0.672	2.781	2.140
Partitioning	9	3	1.038	0.627	0.571	0.454	0.197	0.330	2.332	1.103
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.12	0.183	0.219	0.142	0.169	0.253	0.215	0.180	0.261
B-splines	5	4	0.203	0.199	0.159	0.157	0.428	0.328	0.178	0.187
Partitioning	5	4	0.218	0.254	0.170	0.195	0.316	0.522	0.178	0.222
Feasible Estimation										
Local Polynomial	0.21	0.25	0.181	0.202	0.140	0.157	0.414	0.331	0.181	0.215
B-splines	2	2	0.183	0.190	0.142	0.149	0.427	0.403	0.163	0.171
Partitioning	2	2	0.195	0.190	0.150	0.146	0.413	0.364	0.168	0.177
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.15	0.36	0.148	0.161	0.113	0.121	0.124	0.112	0.184	0.181
B-splines	3	1	0.155	0.158	0.122	0.123	0.147	0.138	0.167	0.174
Partitioning	3	1	0.167	0.158	0.130	0.123	0.122	0.138	0.167	0.174
Feasible Estimation										
Local Polynomial	0.19	0.26	0.145	0.176	0.111	0.133	0.128	0.137	0.180	0.207
B-splines	3	2	0.148	0.145	0.115	0.112	0.161	0.129	0.163	0.170
Partitioning	3	2	0.161	0.179	0.124	0.138	0.167	0.346	0.165	0.171

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.12: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.230	0.249	0.177	0.186	0.325	0.234	0.300	0.401
B-splines	8	5	0.204	0.413	0.158	0.333	0.202	0.931	0.279	0.318
Partitioning	8	5	0.260	0.292	0.200	0.223	0.324	0.259	0.280	0.346
Feasible Estimation										
Local Polynomial	0.1	0.17	0.268	0.270	0.203	0.206	0.666	0.559	0.279	0.382
B-splines	5	3	0.456	0.471	0.356	0.375	1.088	1.141	0.268	0.270
Partitioning	5	3	0.358	0.309	0.265	0.240	0.757	0.547	0.269	0.266
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.229	0.249	0.176	0.186	0.326	0.234	0.300	0.401
B-splines	8	5	0.203	0.413	0.157	0.333	0.202	0.931	0.279	0.316
Partitioning	8	5	0.259	0.292	0.200	0.223	0.324	0.259	0.280	0.346
Feasible Estimation										
Local Polynomial	0.11	0.18	0.287	0.271	0.213	0.206	0.761	0.563	0.277	0.380
B-splines	4	3	0.387	0.469	0.310	0.374	0.843	1.140	0.275	0.267
Partitioning	4	3	0.341	0.308	0.263	0.239	0.645	0.542	0.257	0.266
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.092	0.129	0.071	0.096	0.064	0.088	0.140	0.241
B-splines	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Partitioning	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Feasible Estimation										
Local Polynomial	0.23	0.26	0.134	0.177	0.097	0.128	0.092	0.119	0.266	0.281
B-splines	2	2	0.118	0.141	0.090	0.105	0.107	0.111	0.191	0.254
Partitioning	2	2	0.140	0.177	0.106	0.133	0.143	0.290	0.205	0.259
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.193	0.228	0.126	0.161	0.926	0.866	0.263	0.313
B-splines	5	5	0.211	0.228	0.147	0.161	0.930	0.902	0.263	0.267
Partitioning	5	5	0.244	0.309	0.181	0.235	0.893	0.771	0.269	0.346
Feasible Estimation										
Local Polynomial	0.22	0.25	0.195	0.224	0.127	0.156	0.923	0.887	0.267	0.284
B-splines	2	2	0.186	0.199	0.122	0.135	0.924	0.912	0.200	0.255
Partitioning	2	2	0.200	0.213	0.136	0.154	0.909	0.721	0.213	0.260
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.379	0.266	0.262	0.195	0.105	0.128	0.280	0.316
B-splines	5	5	0.350	0.302	0.282	0.226	0.193	0.162	0.287	0.320
Partitioning	5	5	0.287	0.298	0.212	0.225	0.122	0.183	0.288	0.347
Feasible Estimation										
Local Polynomial	0.1	0.19	0.269	0.267	0.193	0.195	0.126	0.131	0.317	0.337
B-splines	5	2	0.381	0.369	0.297	0.297	0.196	0.279	0.310	0.264
Partitioning	5	2	0.306	0.305	0.223	0.235	0.159	0.318	0.312	0.269
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.03	0.09	0.319	0.259	0.240	0.192	0.244	0.193	0.567	0.413
B-splines	18	5	0.342	0.453	0.237	0.326	0.292	0.324	0.953	1.160
Partitioning	18	5	0.423	0.306	0.310	0.228	0.443	0.183	1.472	0.417
Feasible Estimation										
Local Polynomial	0.06	0.14	0.416	0.261	0.251	0.190	0.174	0.155	1.462	0.428
B-splines	9	3	0.854	0.985	0.412	0.625	0.179	0.319	2.478	1.759
Partitioning	9	3	0.762	0.553	0.376	0.374	0.237	0.182	2.266	1.292
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.12	0.189	0.224	0.143	0.166	0.227	0.189	0.281	0.365
B-splines	6	4	0.182	0.205	0.142	0.161	0.198	0.301	0.273	0.265
Partitioning	6	4	0.225	0.255	0.174	0.194	0.264	0.433	0.277	0.299
Feasible Estimation										
Local Polynomial	0.2	0.24	0.192	0.207	0.148	0.158	0.391	0.303	0.270	0.294
B-splines	3	2	0.198	0.202	0.156	0.160	0.408	0.376	0.213	0.262
Partitioning	3	2	0.207	0.192	0.164	0.146	0.389	0.303	0.226	0.261
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.17	0.37	0.144	0.158	0.106	0.114	0.106	0.101	0.275	0.264
B-splines	3	1	0.149	0.150	0.116	0.112	0.130	0.115	0.231	0.259
Partitioning	3	1	0.164	0.150	0.126	0.112	0.110	0.115	0.231	0.259
Feasible Estimation										
Local Polynomial	0.19	0.25	0.146	0.178	0.107	0.128	0.114	0.119	0.270	0.282
B-splines	3	2	0.142	0.144	0.109	0.107	0.142	0.115	0.218	0.257
Partitioning	3	2	0.157	0.179	0.120	0.136	0.149	0.292	0.223	0.261

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

C.1.2 Quantile Cell Boundaries

Table C.13: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.305	0.219	0.124	0.169
B-splines	9	5	0.270	0.343	0.195	0.251	0.787	1.065	0.134	0.192
Partitioning	9	5	0.229	0.218	0.173	0.167	0.444	0.317	0.192	0.167
Feasible Estimation										
Local Polynomial	0.12	0.19	0.229	0.216	0.159	0.158	0.804	0.655	0.106	0.164
B-splines	4	2	0.350	0.380	0.246	0.274	0.946	1.202	0.108	0.145
Partitioning	4	2	0.287	0.225	0.203	0.160	0.641	0.520	0.106	0.115
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.307	0.219	0.123	0.169
B-splines	9	5	0.269	0.343	0.194	0.251	0.787	1.065	0.134	0.186
Partitioning	9	5	0.228	0.218	0.172	0.167	0.444	0.317	0.192	0.167
Feasible Estimation										
Local Polynomial	0.14	0.19	0.258	0.218	0.167	0.159	0.945	0.664	0.107	0.162
B-splines	3	2	0.393	0.367	0.272	0.266	1.320	1.192	0.116	0.122
Partitioning	3	2	0.353	0.216	0.227	0.154	1.122	0.487	0.105	0.118
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.071	0.045	0.072	0.081	0.102
B-splines	1	1	0.063	0.089	0.049	0.069	0.045	0.072	0.078	0.102
Partitioning	1	1	0.063	0.089	0.049	0.069	0.045	0.072	0.078	0.102
Feasible Estimation										
Local Polynomial	0.3	0.3	0.087	0.122	0.066	0.094	0.072	0.102	0.106	0.130
B-splines	2	2	0.074	0.097	0.057	0.075	0.078	0.090	0.089	0.102
Partitioning	2	2	0.085	0.119	0.065	0.090	0.110	0.192	0.092	0.103
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.142	0.166	0.090	0.119	0.929	0.866	0.108	0.156
B-splines	5	5	0.155	0.166	0.103	0.116	0.941	0.915	0.106	0.134
Partitioning	5	5	0.177	0.221	0.127	0.167	0.904	0.788	0.107	0.167
Feasible Estimation										
Local Polynomial	0.29	0.29	0.140	0.162	0.085	0.112	0.943	0.902	0.106	0.129
B-splines	2	2	0.134	0.146	0.077	0.094	0.948	0.930	0.092	0.102
Partitioning	2	2	0.139	0.151	0.085	0.105	0.911	0.771	0.096	0.104
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.447	0.224	0.332	0.164	0.098	0.120	0.323	0.159
B-splines	5	5	0.293	0.282	0.225	0.222	0.187	0.243	0.273	0.213
Partitioning	5	5	0.262	0.237	0.199	0.181	0.111	0.163	0.247	0.167
Feasible Estimation										
Local Polynomial	0.1	0.23	0.244	0.240	0.181	0.173	0.120	0.119	0.231	0.145
B-splines	5	2	0.295	0.363	0.227	0.288	0.188	0.394	0.273	0.173
Partitioning	5	2	0.263	0.292	0.199	0.225	0.113	0.331	0.248	0.135
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.280	0.207	0.222	0.161	0.297	0.201	0.279	0.185
B-splines	28	7	0.257	0.208	0.201	0.168	0.234	0.212	0.294	0.204
Partitioning	28	7	0.345	0.238	0.268	0.183	0.337	0.196	0.368	0.201
Feasible Estimation										
Local Polynomial	0.06	0.15	0.754	0.359	0.477	0.273	0.158	0.144	1.900	0.484
B-splines	8	3	0.960	1.418	0.553	1.122	0.174	0.677	1.857	2.355
Partitioning	8	3	0.858	0.608	0.491	0.455	0.219	0.151	1.353	0.890
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.08	0.12	0.135	0.159	0.106	0.125	0.216	0.179	0.109	0.164
B-splines	6	4	0.146	0.158	0.115	0.125	0.279	0.349	0.107	0.128
Partitioning	6	4	0.165	0.181	0.129	0.139	0.220	0.262	0.113	0.156
Feasible Estimation										
Local Polynomial	0.25	0.27	0.148	0.155	0.111	0.121	0.439	0.340	0.108	0.132
B-splines	2	2	0.152	0.152	0.116	0.117	0.430	0.407	0.116	0.103
Partitioning	2	2	0.153	0.141	0.115	0.108	0.394	0.282	0.115	0.113
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.12	0.33	0.112	0.121	0.087	0.093	0.109	0.093	0.107	0.125
B-splines	4	1	0.111	0.135	0.087	0.109	0.126	0.144	0.106	0.104
Partitioning	4	1	0.132	0.135	0.102	0.109	0.171	0.144	0.107	0.104
Feasible Estimation										
Local Polynomial	0.2	0.29	0.107	0.123	0.084	0.095	0.110	0.104	0.109	0.130
B-splines	2	2	0.128	0.113	0.102	0.088	0.152	0.113	0.104	0.103
Partitioning	2	2	0.126	0.131	0.098	0.102	0.126	0.205	0.105	0.104

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.14: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.168	0.181	0.132	0.140	0.282	0.199	0.149	0.200
B-splines	9	5	0.240	0.349	0.180	0.266	0.606	0.956	0.127	0.199
Partitioning	9	5	0.216	0.208	0.166	0.160	0.337	0.226	0.138	0.182
Feasible Estimation										
Local Polynomial	0.11	0.18	0.234	0.222	0.167	0.165	0.745	0.610	0.128	0.191
B-splines	4	3	0.331	0.409	0.245	0.307	0.848	1.180	0.129	0.139
Partitioning	4	3	0.268	0.227	0.196	0.167	0.520	0.473	0.128	0.130
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.168	0.181	0.131	0.140	0.284	0.199	0.149	0.200
B-splines	9	5	0.239	0.349	0.179	0.266	0.606	0.956	0.127	0.196
Partitioning	9	5	0.216	0.208	0.166	0.160	0.337	0.226	0.138	0.182
Feasible Estimation										
Local Polynomial	0.13	0.19	0.261	0.224	0.177	0.166	0.871	0.619	0.129	0.189
B-splines	4	3	0.350	0.402	0.257	0.304	0.986	1.173	0.141	0.131
Partitioning	4	3	0.300	0.223	0.207	0.164	0.741	0.455	0.123	0.132
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.051	0.070	0.045	0.068	0.086	0.119
B-splines	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Partitioning	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Feasible Estimation										
Local Polynomial	0.28	0.29	0.089	0.123	0.067	0.093	0.068	0.093	0.126	0.143
B-splines	2	2	0.076	0.098	0.059	0.075	0.076	0.084	0.098	0.120
Partitioning	2	2	0.088	0.120	0.067	0.092	0.107	0.183	0.102	0.121
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.153	0.174	0.094	0.121	0.927	0.864	0.128	0.172
B-splines	5	5	0.164	0.174	0.109	0.120	0.928	0.902	0.126	0.138
Partitioning	5	5	0.184	0.224	0.131	0.170	0.879	0.735	0.127	0.182
Feasible Estimation										
Local Polynomial	0.27	0.28	0.152	0.171	0.091	0.115	0.937	0.897	0.126	0.143
B-splines	2	2	0.145	0.156	0.084	0.100	0.939	0.923	0.102	0.120
Partitioning	2	2	0.150	0.158	0.093	0.110	0.898	0.732	0.107	0.122
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.426	0.229	0.307	0.168	0.092	0.107	0.267	0.176
B-splines	5	5	0.365	0.276	0.299	0.214	0.216	0.178	0.197	0.207
Partitioning	5	5	0.264	0.222	0.198	0.168	0.108	0.161	0.229	0.183
Feasible Estimation										
Local Polynomial	0.1	0.21	0.246	0.237	0.179	0.172	0.106	0.106	0.227	0.168
B-splines	5	2	0.373	0.374	0.304	0.305	0.217	0.358	0.196	0.167
Partitioning	5	2	0.272	0.293	0.202	0.229	0.113	0.322	0.228	0.142
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.268	0.203	0.210	0.155	0.250	0.177	0.320	0.214
B-splines	25	6	0.301	0.291	0.217	0.207	0.224	0.147	0.449	0.499
Partitioning	25	6	0.359	0.238	0.272	0.182	0.324	0.274	0.513	0.205
Feasible Estimation										
Local Polynomial	0.06	0.14	0.632	0.323	0.370	0.240	0.142	0.127	1.833	0.433
B-splines	8	3	1.401	1.394	0.810	1.042	0.178	0.629	2.978	2.342
Partitioning	8	3	1.221	0.706	0.619	0.513	0.215	0.129	2.675	1.297
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.08	0.12	0.139	0.162	0.108	0.125	0.200	0.162	0.131	0.192
B-splines	6	4	0.139	0.160	0.110	0.127	0.220	0.314	0.128	0.140
Partitioning	6	4	0.162	0.181	0.126	0.139	0.197	0.258	0.128	0.159
Feasible Estimation										
Local Polynomial	0.24	0.26	0.157	0.159	0.119	0.124	0.424	0.323	0.130	0.148
B-splines	2	2	0.160	0.162	0.124	0.126	0.414	0.393	0.126	0.123
Partitioning	2	2	0.161	0.142	0.124	0.109	0.373	0.253	0.127	0.129
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.13	0.33	0.110	0.120	0.085	0.091	0.095	0.084	0.128	0.136
B-splines	4	1	0.114	0.131	0.088	0.103	0.123	0.125	0.122	0.129
Partitioning	4	1	0.134	0.131	0.104	0.103	0.166	0.125	0.125	0.129
Feasible Estimation										
Local Polynomial	0.2	0.28	0.107	0.124	0.082	0.094	0.101	0.094	0.129	0.144
B-splines	3	2	0.119	0.111	0.093	0.085	0.124	0.100	0.118	0.123
Partitioning	3	2	0.122	0.130	0.094	0.100	0.109	0.194	0.118	0.123

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.15: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 1$, $X_i \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.173	0.182	0.133	0.136	0.252	0.176	0.231	0.307
B-splines	10	5	0.167	0.340	0.130	0.272	0.235	0.771	0.199	0.213
Partitioning	10	5	0.206	0.202	0.160	0.154	0.256	0.173	0.201	0.212
Feasible Estimation										
Local Polynomial	0.1	0.17	0.236	0.225	0.174	0.170	0.649	0.536	0.205	0.273
B-splines	5	3	0.364	0.443	0.282	0.353	0.876	1.093	0.183	0.223
Partitioning	5	3	0.241	0.195	0.178	0.147	0.473	0.310	0.196	0.193
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.173	0.182	0.132	0.136	0.253	0.176	0.230	0.307
B-splines	10	5	0.166	0.340	0.130	0.272	0.235	0.771	0.199	0.215
Partitioning	10	5	0.205	0.202	0.159	0.154	0.256	0.173	0.201	0.212
Feasible Estimation										
Local Polynomial	0.11	0.17	0.259	0.227	0.185	0.171	0.744	0.542	0.203	0.270
B-splines	4	3	0.288	0.442	0.223	0.352	0.622	1.092	0.195	0.221
Partitioning	4	3	0.221	0.194	0.165	0.147	0.330	0.307	0.171	0.193
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.068	0.045	0.063	0.098	0.173
B-splines	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Partitioning	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Feasible Estimation										
Local Polynomial	0.26	0.28	0.091	0.123	0.066	0.088	0.062	0.082	0.188	0.201
B-splines	2	2	0.078	0.098	0.059	0.073	0.073	0.077	0.124	0.182
Partitioning	2	2	0.091	0.122	0.070	0.091	0.105	0.174	0.131	0.187
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.169	0.186	0.101	0.125	0.923	0.859	0.191	0.226
B-splines	5	5	0.178	0.187	0.117	0.127	0.900	0.877	0.174	0.193
Partitioning	5	5	0.193	0.226	0.136	0.171	0.832	0.644	0.180	0.212
Feasible Estimation										
Local Polynomial	0.24	0.27	0.169	0.184	0.100	0.121	0.926	0.885	0.189	0.204
B-splines	2	2	0.163	0.172	0.096	0.108	0.923	0.910	0.137	0.185
Partitioning	2	2	0.167	0.167	0.105	0.116	0.873	0.678	0.147	0.190
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.367	0.230	0.251	0.164	0.083	0.093	0.224	0.229
B-splines	5	5	0.474	0.248	0.346	0.177	0.147	0.097	0.379	0.201
Partitioning	5	5	0.399	0.234	0.273	0.178	0.107	0.161	0.319	0.221
Feasible Estimation										
Local Polynomial	0.1	0.2	0.245	0.229	0.169	0.164	0.090	0.094	0.256	0.238
B-splines	5	2	0.497	0.368	0.356	0.300	0.148	0.279	0.371	0.204
Partitioning	5	2	0.435	0.282	0.293	0.219	0.138	0.257	0.347	0.203
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.240	0.190	0.181	0.140	0.193	0.144	0.443	0.323
B-splines	21	6	0.649	0.464	0.325	0.321	0.204	0.225	2.148	1.218
Partitioning	21	6	0.609	0.296	0.341	0.217	0.289	0.283	1.888	0.591
Feasible Estimation										
Local Polynomial	0.06	0.14	0.419	0.257	0.223	0.180	0.125	0.111	1.649	0.403
B-splines	8	3	1.531	1.086	0.788	0.698	0.165	0.406	1.652	1.720
Partitioning	8	3	1.406	0.684	0.643	0.460	0.213	0.122	1.892	1.534
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.142	0.163	0.108	0.121	0.180	0.143	0.208	0.276
B-splines	6	4	0.127	0.157	0.100	0.125	0.156	0.251	0.183	0.195
Partitioning	6	4	0.158	0.180	0.122	0.137	0.195	0.254	0.188	0.200
Feasible Estimation										
Local Polynomial	0.21	0.24	0.166	0.162	0.128	0.124	0.392	0.289	0.195	0.212
B-splines	2	2	0.173	0.174	0.137	0.140	0.392	0.365	0.147	0.196
Partitioning	2	2	0.168	0.141	0.132	0.107	0.327	0.215	0.167	0.192
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.15	0.34	0.108	0.116	0.079	0.084	0.082	0.074	0.206	0.195
B-splines	3	1	0.108	0.119	0.082	0.090	0.094	0.098	0.162	0.202
Partitioning	3	1	0.119	0.119	0.092	0.090	0.086	0.098	0.160	0.202
Feasible Estimation										
Local Polynomial	0.19	0.27	0.106	0.124	0.079	0.089	0.090	0.084	0.197	0.203
B-splines	3	2	0.103	0.106	0.078	0.079	0.093	0.085	0.152	0.187
Partitioning	3	2	0.113	0.127	0.086	0.096	0.099	0.181	0.151	0.192

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.16: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 1.1					
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.290	0.330	0.226	0.258	0.494	0.383	0.223	0.333
B-splines	7	4	0.396	0.369	0.299	0.288	1.054	0.938	0.215	0.275
Partitioning	7	4	0.381	0.366	0.292	0.282	0.674	0.546	0.256	0.312
Feasible Estimation										
Local Polynomial	0.13	0.2	0.310	0.325	0.232	0.251	0.885	0.744	0.214	0.315
B-splines	4	2	0.412	0.418	0.306	0.314	1.097	1.201	0.220	0.230
Partitioning	4	2	0.383	0.318	0.284	0.240	0.895	0.568	0.212	0.225
					Model 1.2					
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.289	0.330	0.225	0.258	0.497	0.383	0.222	0.333
B-splines	7	4	0.394	0.368	0.298	0.288	1.054	0.938	0.215	0.262
Partitioning	7	4	0.380	0.366	0.291	0.282	0.674	0.546	0.256	0.312
Feasible Estimation										
Local Polynomial	0.16	0.21	0.330	0.326	0.238	0.251	1.034	0.753	0.214	0.312
B-splines	3	2	0.436	0.405	0.315	0.305	1.366	1.199	0.217	0.215
Partitioning	3	2	0.415	0.315	0.295	0.238	1.184	0.567	0.213	0.226
					Model 1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.129	0.181	0.101	0.141	0.090	0.143	0.163	0.203
B-splines	1	1	0.126	0.178	0.098	0.139	0.090	0.143	0.156	0.203
Partitioning	1	1	0.126	0.178	0.098	0.139	0.090	0.143	0.156	0.203
Feasible Estimation										
Local Polynomial	0.24	0.26	0.184	0.249	0.140	0.192	0.161	0.216	0.215	0.268
B-splines	2	2	0.160	0.197	0.124	0.153	0.160	0.186	0.188	0.204
Partitioning	2	2	0.192	0.248	0.146	0.190	0.225	0.411	0.194	0.208
					Model 1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.214	0.283	0.157	0.217	0.940	0.889	0.215	0.312
B-splines	5	5	0.245	0.274	0.185	0.210	0.952	0.931	0.211	0.267
Partitioning	5	5	0.303	0.411	0.233	0.316	0.921	0.836	0.214	0.335
Feasible Estimation										
Local Polynomial	0.24	0.26	0.214	0.270	0.156	0.206	0.943	0.912	0.215	0.269
B-splines	2	2	0.195	0.225	0.140	0.167	0.957	0.942	0.189	0.204
Partitioning	2	2	0.221	0.264	0.161	0.199	0.928	0.824	0.196	0.208
					Model 1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.474	0.321	0.361	0.249	0.172	0.239	0.374	0.313
B-splines	5	5	0.349	0.357	0.273	0.283	0.237	0.299	0.329	0.315
Partitioning	5	5	0.359	0.420	0.279	0.324	0.216	0.325	0.310	0.335
Feasible Estimation										
Local Polynomial	0.1	0.24	0.326	0.335	0.252	0.259	0.232	0.232	0.299	0.285
B-splines	5	2	0.390	0.417	0.309	0.330	0.263	0.426	0.323	0.245
Partitioning	5	2	0.361	0.384	0.279	0.297	0.269	0.487	0.311	0.227
					Model 1.6					
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.486	0.377	0.385	0.295	0.507	0.372	0.484	0.347
B-splines	21	6	0.445	0.304	0.347	0.237	0.407	0.277	0.495	0.333
Partitioning	21	6	0.595	0.440	0.463	0.338	0.568	0.544	0.652	0.354
Feasible Estimation										
Local Polynomial	0.06	0.15	0.810	0.451	0.559	0.354	0.305	0.283	1.955	0.629
B-splines	8	3	0.989	1.430	0.612	1.129	0.305	0.693	1.872	2.362
Partitioning	8	3	0.914	0.665	0.586	0.511	0.424	0.259	1.383	0.916
					Model 1.7					
Infeasible Estimation										
Local Polynomial	0.11	0.14	0.237	0.299	0.185	0.233	0.332	0.315	0.214	0.316
B-splines	4	3	0.225	0.249	0.177	0.196	0.401	0.452	0.212	0.207
Partitioning	4	3	0.268	0.320	0.209	0.247	0.396	0.352	0.212	0.255
Feasible Estimation										
Local Polynomial	0.23	0.26	0.218	0.266	0.169	0.208	0.445	0.379	0.216	0.271
B-splines	2	2	0.208	0.229	0.163	0.180	0.458	0.437	0.199	0.204
Partitioning	2	2	0.230	0.258	0.177	0.199	0.443	0.444	0.203	0.213
					Model 1.8					
Infeasible Estimation										
Local Polynomial	0.16	0.38	0.202	0.222	0.157	0.171	0.187	0.170	0.218	0.221
B-splines	3	1	0.207	0.205	0.165	0.161	0.205	0.190	0.205	0.204
Partitioning	3	1	0.229	0.205	0.178	0.161	0.177	0.190	0.208	0.204
Feasible Estimation										
Local Polynomial	0.21	0.26	0.197	0.250	0.153	0.193	0.188	0.217	0.215	0.268
B-splines	2	2	0.196	0.201	0.154	0.157	0.217	0.193	0.201	0.204
Partitioning	2	2	0.217	0.251	0.168	0.193	0.236	0.414	0.205	0.208

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.17: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.295	0.335	0.230	0.259	0.458	0.347	0.268	0.390
B-splines	7	4	0.380	0.370	0.295	0.294	0.882	0.832	0.255	0.281
Partitioning	7	4	0.371	0.365	0.287	0.281	0.532	0.530	0.258	0.318
Feasible Estimation										
Local Polynomial	0.12	0.2	0.321	0.334	0.243	0.257	0.844	0.705	0.257	0.356
B-splines	4	2	0.385	0.443	0.296	0.342	0.900	1.157	0.265	0.251
Partitioning	4	2	0.361	0.319	0.275	0.243	0.670	0.476	0.251	0.256
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.294	0.335	0.229	0.259	0.460	0.347	0.268	0.390
B-splines	7	4	0.379	0.370	0.294	0.294	0.882	0.833	0.255	0.277
Partitioning	7	4	0.370	0.365	0.286	0.281	0.532	0.530	0.258	0.318
Feasible Estimation										
Local Polynomial	0.15	0.2	0.343	0.335	0.252	0.258	0.977	0.712	0.257	0.353
B-splines	3	2	0.438	0.433	0.328	0.335	1.195	1.155	0.262	0.246
Partitioning	3	2	0.404	0.317	0.294	0.241	0.988	0.474	0.246	0.257
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.130	0.183	0.101	0.140	0.090	0.136	0.171	0.238
B-splines	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Partitioning	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Feasible Estimation										
Local Polynomial	0.23	0.26	0.188	0.250	0.141	0.189	0.148	0.194	0.256	0.296
B-splines	2	2	0.163	0.199	0.126	0.153	0.158	0.173	0.209	0.241
Partitioning	2	2	0.194	0.249	0.149	0.191	0.218	0.387	0.217	0.243
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.222	0.288	0.160	0.217	0.937	0.884	0.256	0.344
B-splines	5	5	0.252	0.281	0.190	0.212	0.940	0.918	0.252	0.274
Partitioning	5	5	0.307	0.414	0.236	0.319	0.899	0.786	0.254	0.363
Feasible Estimation										
Local Polynomial	0.23	0.26	0.224	0.277	0.160	0.206	0.937	0.907	0.256	0.298
B-splines	2	2	0.205	0.233	0.146	0.171	0.948	0.934	0.211	0.241
Partitioning	2	2	0.228	0.268	0.167	0.202	0.913	0.780	0.219	0.244
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.455	0.324	0.337	0.249	0.158	0.214	0.347	0.347
B-splines	5	5	0.412	0.353	0.333	0.276	0.260	0.244	0.295	0.315
Partitioning	5	5	0.361	0.413	0.277	0.317	0.214	0.322	0.317	0.363
Feasible Estimation										
Local Polynomial	0.11	0.23	0.329	0.334	0.251	0.256	0.203	0.210	0.319	0.321
B-splines	5	2	0.485	0.417	0.385	0.336	0.280	0.388	0.288	0.268
Partitioning	5	2	0.405	0.375	0.308	0.293	0.279	0.455	0.308	0.258
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.09	0.463	0.370	0.363	0.285	0.433	0.327	0.550	0.407
B-splines	19	5	0.512	0.599	0.375	0.460	0.383	0.459	0.782	1.107
Partitioning	19	5	0.624	0.433	0.475	0.332	0.551	0.322	0.948	0.410
Feasible Estimation										
Local Polynomial	0.06	0.15	0.700	0.424	0.459	0.324	0.271	0.249	1.916	0.628
B-splines	8	3	1.435	1.407	0.864	1.050	0.311	0.644	2.995	2.353
Partitioning	8	3	1.273	0.756	0.730	0.567	0.416	0.244	2.701	1.318
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.1	0.13	0.244	0.303	0.188	0.233	0.311	0.284	0.257	0.365
B-splines	5	4	0.255	0.261	0.202	0.204	0.433	0.363	0.253	0.258
Partitioning	5	4	0.296	0.360	0.231	0.277	0.335	0.503	0.254	0.318
Feasible Estimation										
Local Polynomial	0.22	0.25	0.226	0.270	0.175	0.208	0.432	0.363	0.259	0.301
B-splines	2	2	0.217	0.237	0.170	0.186	0.444	0.422	0.223	0.242
Partitioning	2	2	0.237	0.260	0.183	0.200	0.420	0.414	0.230	0.247
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.17	0.39	0.200	0.221	0.153	0.167	0.165	0.155	0.260	0.251
B-splines	3	1	0.200	0.204	0.157	0.157	0.175	0.171	0.232	0.242
Partitioning	3	1	0.229	0.204	0.178	0.157	0.168	0.171	0.236	0.242
Feasible Estimation										
Local Polynomial	0.21	0.26	0.199	0.251	0.151	0.189	0.170	0.195	0.257	0.297
B-splines	2	2	0.192	0.203	0.149	0.156	0.193	0.178	0.225	0.242
Partitioning	2	2	0.217	0.252	0.167	0.193	0.218	0.391	0.232	0.244

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.18: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

$d = 1$, $n = 500$, $\sigma^2 = 4$, $X_i \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 1.1					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.303	0.338	0.230	0.251	0.410	0.307	0.422	0.573
B-splines	7	5	0.335	0.404	0.266	0.323	0.614	0.786	0.378	0.395
Partitioning	7	5	0.354	0.401	0.275	0.306	0.386	0.328	0.385	0.425
Feasible Estimation										
Local Polynomial	0.12	0.19	0.336	0.342	0.255	0.260	0.781	0.640	0.406	0.493
B-splines	4	3	0.352	0.473	0.279	0.377	0.647	1.068	0.373	0.410
Partitioning	4	3	0.327	0.309	0.253	0.235	0.440	0.354	0.367	0.384
					Model 1.2					
Infeasible Estimation										
Local Polynomial	0.07	0.1	0.302	0.338	0.230	0.251	0.412	0.307	0.422	0.573
B-splines	7	5	0.334	0.404	0.265	0.323	0.614	0.786	0.377	0.395
Partitioning	7	5	0.353	0.401	0.274	0.306	0.386	0.328	0.385	0.425
Feasible Estimation										
Local Polynomial	0.13	0.19	0.356	0.342	0.266	0.261	0.883	0.644	0.401	0.491
B-splines	4	3	0.405	0.467	0.310	0.374	0.898	1.067	0.375	0.396
Partitioning	4	3	0.354	0.308	0.263	0.234	0.680	0.354	0.331	0.384
					Model 1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.129	0.182	0.100	0.136	0.090	0.125	0.197	0.346
B-splines	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Partitioning	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Feasible Estimation										
Local Polynomial	0.22	0.26	0.190	0.250	0.137	0.179	0.133	0.170	0.388	0.411
B-splines	2	2	0.165	0.199	0.126	0.148	0.149	0.158	0.265	0.366
Partitioning	2	2	0.196	0.249	0.150	0.187	0.207	0.360	0.281	0.377
					Model 1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.234	0.294	0.162	0.212	0.930	0.874	0.382	0.453
B-splines	5	5	0.261	0.288	0.195	0.214	0.911	0.890	0.347	0.385
Partitioning	5	5	0.311	0.414	0.239	0.317	0.851	0.703	0.359	0.425
Feasible Estimation										
Local Polynomial	0.22	0.25	0.238	0.285	0.164	0.204	0.927	0.893	0.389	0.414
B-splines	2	2	0.219	0.244	0.155	0.175	0.928	0.918	0.272	0.368
Partitioning	2	2	0.241	0.274	0.175	0.204	0.887	0.733	0.289	0.380
					Model 1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.402	0.323	0.283	0.238	0.139	0.185	0.401	0.454
B-splines	5	5	0.511	0.331	0.381	0.247	0.205	0.186	0.483	0.389
Partitioning	5	5	0.468	0.418	0.348	0.321	0.214	0.322	0.445	0.429
Feasible Estimation										
Local Polynomial	0.12	0.21	0.328	0.328	0.239	0.242	0.170	0.184	0.436	0.451
B-splines	4	2	0.553	0.413	0.397	0.329	0.231	0.313	0.459	0.385
Partitioning	4	2	0.526	0.367	0.381	0.285	0.302	0.398	0.471	0.394
					Model 1.6					
Infeasible Estimation										
Local Polynomial	0.03	0.1	0.418	0.350	0.314	0.259	0.333	0.266	0.756	0.594
B-splines	16	5	0.917	0.700	0.520	0.499	0.371	0.425	2.435	1.584
Partitioning	16	5	0.906	0.490	0.565	0.366	0.540	0.322	2.325	0.892
Feasible Estimation										
Local Polynomial	0.06	0.15	0.515	0.371	0.318	0.268	0.231	0.215	1.810	0.703
B-splines	7	3	1.639	1.103	0.885	0.710	0.273	0.427	1.506	1.746
Partitioning	7	3	1.530	0.734	0.792	0.516	0.309	0.243	1.743	1.566
					Model 1.7					
Infeasible Estimation										
Local Polynomial	0.1	0.13	0.250	0.305	0.187	0.225	0.282	0.252	0.409	0.511
B-splines	5	4	0.256	0.259	0.203	0.198	0.368	0.304	0.354	0.384
Partitioning	5	4	0.290	0.359	0.226	0.273	0.274	0.504	0.360	0.400
Feasible Estimation										
Local Polynomial	0.21	0.24	0.236	0.273	0.181	0.204	0.407	0.333	0.391	0.420
B-splines	2	2	0.228	0.246	0.180	0.193	0.413	0.393	0.280	0.374
Partitioning	2	2	0.243	0.262	0.189	0.198	0.373	0.381	0.301	0.380
					Model 1.8					
Infeasible Estimation										
Local Polynomial	0.19	0.4	0.193	0.215	0.142	0.156	0.142	0.139	0.386	0.386
B-splines	2	1	0.162	0.196	0.124	0.146	0.162	0.146	0.255	0.357
Partitioning	2	1	0.185	0.196	0.143	0.146	0.233	0.146	0.268	0.357
Feasible Estimation										
Local Polynomial	0.2	0.25	0.200	0.250	0.146	0.180	0.152	0.171	0.392	0.412
B-splines	3	2	0.183	0.202	0.139	0.151	0.167	0.161	0.286	0.367
Partitioning	3	2	0.212	0.251	0.162	0.190	0.208	0.364	0.296	0.379

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.19: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 1.1					
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.125	0.132	0.097	0.103	0.240	0.168	0.093	0.118
B-splines	10	5	0.151	0.335	0.112	0.242	0.323	1.073	0.111	0.166
Partitioning	10	5	0.155	0.168	0.120	0.127	0.209	0.309	0.165	0.117
Feasible Estimation										
Local Polynomial	0.1	0.18	0.187	0.178	0.126	0.126	0.686	0.576	0.075	0.127
B-splines	5	3	0.386	0.382	0.259	0.271	1.243	1.275	0.075	0.119
Partitioning	5	3	0.300	0.254	0.187	0.172	0.906	0.737	0.075	0.089
					Model 1.2					
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.124	0.132	0.097	0.103	0.242	0.168	0.093	0.118
B-splines	10	5	0.150	0.335	0.111	0.242	0.323	1.073	0.111	0.160
Partitioning	10	5	0.154	0.168	0.119	0.127	0.208	0.309	0.165	0.117
Feasible Estimation										
Local Polynomial	0.12	0.18	0.215	0.180	0.134	0.127	0.822	0.585	0.075	0.125
B-splines	4	3	0.307	0.381	0.223	0.270	0.939	1.272	0.076	0.112
Partitioning	4	3	0.270	0.252	0.189	0.170	0.671	0.726	0.075	0.089
					Model 1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.065	0.036	0.050	0.032	0.051	0.057	0.072
B-splines	1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Partitioning	1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Feasible Estimation										
Local Polynomial	0.3	0.3	0.062	0.087	0.047	0.067	0.051	0.072	0.075	0.090
B-splines	2	2	0.053	0.069	0.041	0.054	0.056	0.065	0.063	0.071
Partitioning	2	2	0.061	0.084	0.047	0.064	0.079	0.143	0.065	0.073
					Model 1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.127	0.138	0.071	0.091	0.927	0.864	0.076	0.109
B-splines	5	5	0.135	0.140	0.081	0.090	0.942	0.914	0.074	0.096
Partitioning	5	5	0.147	0.171	0.097	0.125	0.907	0.790	0.075	0.117
Feasible Estimation										
Local Polynomial	0.28	0.28	0.126	0.137	0.069	0.087	0.939	0.898	0.075	0.092
B-splines	2	2	0.123	0.129	0.063	0.075	0.944	0.926	0.066	0.072
Partitioning	2	2	0.125	0.125	0.069	0.082	0.908	0.725	0.069	0.074
					Model 1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.443	0.205	0.327	0.143	0.079	0.084	0.315	0.114
B-splines	5	5	0.277	0.269	0.211	0.212	0.177	0.236	0.267	0.194
Partitioning	5	5	0.240	0.193	0.180	0.144	0.079	0.112	0.237	0.117
Feasible Estimation										
Local Polynomial	0.09	0.21	0.207	0.210	0.150	0.146	0.091	0.084	0.202	0.112
B-splines	6	2	0.287	0.353	0.211	0.280	0.131	0.390	0.228	0.158
Partitioning	6	2	0.249	0.271	0.178	0.210	0.126	0.311	0.209	0.114
					Model 1.6					
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.211	0.144	0.168	0.113	0.223	0.149	0.205	0.131
B-splines	32	7	0.192	0.183	0.150	0.152	0.179	0.204	0.207	0.172
Partitioning	32	7	0.259	0.169	0.201	0.130	0.257	0.136	0.268	0.139
Feasible Estimation										
Local Polynomial	0.05	0.13	0.618	0.228	0.387	0.172	0.121	0.106	1.469	0.120
B-splines	9	4	0.735	0.881	0.415	0.666	0.156	0.586	1.063	1.465
Partitioning	9	4	0.670	0.345	0.385	0.231	0.127	0.177	0.706	0.438
					Model 1.7					
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.103	0.117	0.080	0.092	0.173	0.137	0.079	0.116
B-splines	7	4	0.142	0.135	0.108	0.105	0.377	0.337	0.078	0.099
Partitioning	7	4	0.135	0.129	0.104	0.099	0.238	0.205	0.089	0.110
Feasible Estimation										
Local Polynomial	0.23	0.25	0.133	0.126	0.097	0.097	0.424	0.318	0.077	0.095
B-splines	2	2	0.140	0.136	0.106	0.102	0.416	0.397	0.085	0.073
Partitioning	2	2	0.139	0.108	0.104	0.083	0.383	0.214	0.083	0.086
					Model 1.8					
Infeasible Estimation										
Local Polynomial	0.11	0.3	0.084	0.091	0.066	0.070	0.082	0.069	0.076	0.097
B-splines	4	2	0.087	0.076	0.068	0.059	0.107	0.072	0.076	0.072
Partitioning	4	2	0.097	0.090	0.076	0.070	0.131	0.163	0.076	0.073
Feasible Estimation										
Local Polynomial	0.17	0.29	0.082	0.088	0.064	0.068	0.082	0.074	0.078	0.090
B-splines	3	2	0.118	0.089	0.097	0.070	0.156	0.092	0.074	0.073
Partitioning	3	2	0.102	0.100	0.080	0.078	0.101	0.161	0.074	0.074

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.20: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.220	0.151	0.111	0.139
B-splines	11	6	0.175	0.214	0.131	0.168	0.449	0.521	0.093	0.137
Partitioning	11	6	0.164	0.157	0.126	0.121	0.240	0.220	0.115	0.136
Feasible Estimation										
Local Polynomial	0.1	0.17	0.190	0.181	0.132	0.131	0.631	0.536	0.090	0.150
B-splines	5	3	0.401	0.407	0.283	0.304	1.199	1.214	0.092	0.110
Partitioning	5	3	0.275	0.224	0.173	0.158	0.788	0.564	0.089	0.093
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.221	0.151	0.111	0.139
B-splines	10	6	0.134	0.214	0.102	0.168	0.232	0.521	0.090	0.138
Partitioning	10	6	0.149	0.157	0.116	0.121	0.207	0.220	0.103	0.136
Feasible Estimation										
Local Polynomial	0.11	0.17	0.216	0.183	0.141	0.132	0.750	0.544	0.089	0.148
B-splines	4	3	0.309	0.407	0.231	0.304	0.831	1.213	0.096	0.102
Partitioning	4	3	0.249	0.224	0.175	0.158	0.516	0.563	0.088	0.093
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.049	0.031	0.047	0.061	0.084
B-splines	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Partitioning	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Feasible Estimation										
Local Polynomial	0.28	0.29	0.062	0.087	0.047	0.065	0.047	0.065	0.088	0.099
B-splines	2	2	0.053	0.069	0.041	0.053	0.053	0.059	0.069	0.084
Partitioning	2	2	0.062	0.085	0.047	0.065	0.075	0.134	0.072	0.085
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.137	0.147	0.075	0.094	0.927	0.863	0.089	0.119
B-splines	5	5	0.144	0.149	0.085	0.094	0.929	0.902	0.088	0.097
Partitioning	5	5	0.154	0.174	0.100	0.127	0.886	0.740	0.089	0.126
Feasible Estimation										
Local Polynomial	0.26	0.28	0.137	0.146	0.073	0.090	0.935	0.894	0.088	0.101
B-splines	2	2	0.134	0.140	0.069	0.080	0.938	0.922	0.075	0.085
Partitioning	2	2	0.136	0.131	0.076	0.086	0.896	0.695	0.079	0.086
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.421	0.210	0.301	0.147	0.075	0.076	0.251	0.125
B-splines	5	5	0.354	0.263	0.295	0.203	0.213	0.168	0.171	0.186
Partitioning	5	5	0.243	0.169	0.181	0.125	0.076	0.109	0.211	0.126
Feasible Estimation										
Local Polynomial	0.09	0.2	0.209	0.209	0.147	0.146	0.080	0.077	0.195	0.129
B-splines	6	2	0.300	0.356	0.230	0.290	0.148	0.340	0.206	0.143
Partitioning	6	2	0.232	0.267	0.172	0.207	0.115	0.269	0.219	0.115
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.07	0.201	0.141	0.158	0.110	0.184	0.128	0.237	0.151
B-splines	29	7	0.221	0.168	0.162	0.121	0.169	0.093	0.278	0.195
Partitioning	29	7	0.272	0.174	0.207	0.134	0.240	0.135	0.301	0.150
Feasible Estimation										
Local Polynomial	0.05	0.13	0.517	0.209	0.298	0.155	0.107	0.094	1.430	0.152
B-splines	9	4	1.170	0.936	0.635	0.718	0.099	0.620	2.740	1.735
Partitioning	9	4	1.017	0.415	0.493	0.280	0.114	0.175	2.300	0.704
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.104	0.118	0.081	0.091	0.160	0.123	0.093	0.135
B-splines	7	4	0.139	0.136	0.109	0.109	0.322	0.303	0.090	0.109
Partitioning	7	4	0.130	0.128	0.101	0.099	0.188	0.198	0.090	0.111
Feasible Estimation										
Local Polynomial	0.21	0.24	0.140	0.128	0.103	0.099	0.408	0.300	0.091	0.106
B-splines	2	2	0.149	0.145	0.114	0.112	0.412	0.384	0.092	0.088
Partitioning	2	2	0.148	0.108	0.112	0.083	0.369	0.184	0.093	0.096
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.11	0.31	0.082	0.089	0.063	0.067	0.071	0.061	0.090	0.102
B-splines	4	2	0.089	0.075	0.069	0.058	0.102	0.065	0.086	0.085
Partitioning	4	2	0.099	0.090	0.076	0.069	0.124	0.151	0.087	0.085
Feasible Estimation										
Local Polynomial	0.17	0.28	0.080	0.087	0.062	0.066	0.075	0.067	0.091	0.100
B-splines	3	2	0.105	0.085	0.084	0.065	0.121	0.077	0.088	0.088
Partitioning	3	2	0.097	0.097	0.075	0.075	0.082	0.146	0.084	0.088

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.21: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 1$, $X_i \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.04	0.08	0.132	0.136	0.102	0.101	0.196	0.134	0.169	0.209
B-splines	11	6	0.132	0.166	0.103	0.133	0.244	0.302	0.138	0.149
Partitioning	11	6	0.155	0.156	0.120	0.119	0.169	0.212	0.139	0.155
Feasible Estimation										
Local Polynomial	0.09	0.16	0.191	0.185	0.139	0.138	0.545	0.470	0.140	0.218
B-splines	5	3	0.310	0.439	0.218	0.350	0.777	1.100	0.120	0.154
Partitioning	5	3	0.197	0.166	0.137	0.123	0.443	0.314	0.139	0.132
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.04	0.08	0.131	0.136	0.101	0.101	0.197	0.134	0.169	0.209
B-splines	11	6	0.131	0.166	0.102	0.133	0.244	0.302	0.138	0.148
Partitioning	11	6	0.154	0.156	0.120	0.119	0.169	0.212	0.139	0.155
Feasible Estimation										
Local Polynomial	0.1	0.16	0.212	0.187	0.149	0.139	0.633	0.476	0.139	0.216
B-splines	5	3	0.365	0.438	0.279	0.350	0.924	1.100	0.152	0.157
Partitioning	5	3	0.219	0.166	0.150	0.123	0.505	0.314	0.124	0.131
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.048	0.032	0.044	0.070	0.120
B-splines	1	1	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Partitioning	1	1	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Feasible Estimation										
Local Polynomial	0.26	0.28	0.064	0.087	0.046	0.063	0.044	0.057	0.129	0.137
B-splines	2	2	0.055	0.069	0.042	0.052	0.052	0.054	0.087	0.126
Partitioning	2	2	0.064	0.086	0.049	0.065	0.074	0.128	0.093	0.128
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.156	0.162	0.084	0.101	0.923	0.858	0.131	0.156
B-splines	5	5	0.160	0.164	0.096	0.103	0.902	0.878	0.121	0.131
Partitioning	5	5	0.166	0.178	0.109	0.131	0.840	0.648	0.125	0.145
Feasible Estimation										
Local Polynomial	0.23	0.26	0.156	0.162	0.084	0.099	0.923	0.883	0.131	0.140
B-splines	2	2	0.154	0.157	0.083	0.091	0.921	0.907	0.102	0.128
Partitioning	2	2	0.154	0.142	0.090	0.095	0.873	0.643	0.108	0.130
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.361	0.211	0.244	0.146	0.069	0.065	0.162	0.163
B-splines	5	5	0.471	0.233	0.342	0.162	0.134	0.069	0.343	0.144
Partitioning	5	5	0.391	0.187	0.259	0.139	0.073	0.110	0.272	0.155
Feasible Estimation										
Local Polynomial	0.09	0.18	0.210	0.203	0.140	0.140	0.067	0.068	0.203	0.183
B-splines	5	3	0.465	0.330	0.337	0.270	0.129	0.249	0.338	0.148
Partitioning	5	3	0.384	0.239	0.253	0.175	0.080	0.125	0.269	0.137
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.184	0.141	0.138	0.104	0.142	0.104	0.329	0.220
B-splines	24	6	0.548	0.464	0.258	0.323	0.156	0.224	1.883	1.220
Partitioning	24	6	0.512	0.254	0.270	0.179	0.226	0.211	1.553	0.548
Feasible Estimation										
Local Polynomial	0.05	0.13	0.343	0.169	0.178	0.120	0.094	0.081	1.306	0.244
B-splines	9	4	1.436	0.878	0.716	0.607	0.080	0.488	1.826	1.710
Partitioning	9	4	1.305	0.511	0.561	0.329	0.107	0.172	2.073	1.250
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.107	0.120	0.082	0.090	0.144	0.109	0.144	0.195
B-splines	7	5	0.122	0.148	0.097	0.120	0.225	0.289	0.130	0.138
Partitioning	7	5	0.125	0.142	0.098	0.109	0.135	0.113	0.133	0.145
Feasible Estimation										
Local Polynomial	0.18	0.22	0.147	0.130	0.112	0.100	0.373	0.264	0.135	0.151
B-splines	3	2	0.166	0.159	0.130	0.128	0.399	0.360	0.111	0.143
Partitioning	3	2	0.153	0.106	0.118	0.081	0.325	0.147	0.135	0.132
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.13	0.32	0.080	0.087	0.059	0.062	0.061	0.054	0.137	0.136
B-splines	4	2	0.085	0.074	0.064	0.055	0.089	0.058	0.119	0.128
Partitioning	4	2	0.098	0.090	0.075	0.068	0.119	0.140	0.121	0.130
Feasible Estimation										
Local Polynomial	0.16	0.27	0.079	0.088	0.059	0.064	0.068	0.059	0.137	0.138
B-splines	3	2	0.086	0.079	0.065	0.058	0.086	0.063	0.120	0.133
Partitioning	3	2	0.090	0.093	0.068	0.070	0.075	0.135	0.113	0.135

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.22: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		Point Estimation RMSE (0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.218	0.243	0.170	0.190	0.394	0.293	0.164	0.234
B-splines	8	5	0.239	0.370	0.186	0.278	0.448	1.081	0.163	0.233
Partitioning	8	5	0.270	0.297	0.210	0.229	0.333	0.366	0.217	0.235
Feasible Estimation										
Local Polynomial	0.11	0.19	0.247	0.254	0.181	0.193	0.761	0.645	0.150	0.241
B-splines	4	3	0.377	0.400	0.273	0.294	1.035	1.243	0.147	0.176
Partitioning	4	3	0.328	0.289	0.238	0.215	0.754	0.638	0.150	0.167
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.217	0.243	0.170	0.190	0.396	0.293	0.163	0.234
B-splines	8	5	0.238	0.369	0.184	0.278	0.447	1.081	0.163	0.229
Partitioning	8	5	0.269	0.297	0.209	0.229	0.331	0.366	0.217	0.235
Feasible Estimation										
Local Polynomial	0.13	0.19	0.271	0.255	0.189	0.194	0.907	0.653	0.151	0.239
B-splines	4	3	0.371	0.392	0.271	0.288	1.140	1.238	0.153	0.164
Partitioning	4	3	0.348	0.285	0.246	0.211	0.955	0.620	0.149	0.167
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.093	0.129	0.072	0.101	0.064	0.102	0.115	0.143
B-splines	1	1	0.090	0.127	0.070	0.099	0.063	0.102	0.109	0.143
Partitioning	1	1	0.090	0.127	0.070	0.099	0.063	0.102	0.109	0.143
Feasible Estimation										
Local Polynomial	0.24	0.27	0.132	0.177	0.101	0.137	0.114	0.151	0.151	0.188
B-splines	2	2	0.114	0.140	0.089	0.110	0.118	0.134	0.133	0.143
Partitioning	2	2	0.137	0.176	0.105	0.135	0.160	0.307	0.136	0.147
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.171	0.214	0.118	0.160	0.933	0.877	0.152	0.218
B-splines	5	5	0.191	0.209	0.138	0.155	0.948	0.922	0.149	0.190
Partitioning	5	5	0.228	0.299	0.171	0.229	0.916	0.814	0.151	0.235
Feasible Estimation										
Local Polynomial	0.24	0.26	0.171	0.206	0.118	0.152	0.935	0.900	0.151	0.190
B-splines	2	2	0.160	0.178	0.107	0.126	0.949	0.931	0.134	0.143
Partitioning	2	2	0.175	0.197	0.121	0.146	0.917	0.749	0.138	0.147
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.457	0.261	0.344	0.199	0.128	0.168	0.342	0.221
B-splines	5	5	0.308	0.310	0.239	0.247	0.204	0.267	0.298	0.254
Partitioning	5	5	0.297	0.312	0.229	0.241	0.148	0.223	0.272	0.235
Feasible Estimation										
Local Polynomial	0.09	0.22	0.264	0.272	0.201	0.206	0.176	0.168	0.246	0.212
B-splines	5	2	0.312	0.375	0.241	0.297	0.199	0.403	0.286	0.200
Partitioning	5	2	0.301	0.317	0.231	0.245	0.186	0.391	0.263	0.175
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.368	0.279	0.293	0.218	0.385	0.276	0.361	0.247
B-splines	24	6	0.334	0.228	0.261	0.179	0.311	0.205	0.390	0.243
Partitioning	24	6	0.449	0.312	0.349	0.240	0.444	0.424	0.529	0.248
Feasible Estimation										
Local Polynomial	0.05	0.14	0.659	0.294	0.450	0.233	0.233	0.208	1.521	0.229
B-splines	9	3	0.755	1.198	0.459	0.912	0.209	0.664	1.075	1.975
Partitioning	9	3	0.709	0.528	0.454	0.387	0.223	0.276	0.742	0.721
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.13	0.180	0.218	0.141	0.171	0.271	0.239	0.152	0.226
B-splines	5	4	0.199	0.198	0.155	0.156	0.451	0.364	0.149	0.171
Partitioning	5	4	0.222	0.254	0.173	0.196	0.363	0.389	0.151	0.220
Feasible Estimation										
Local Polynomial	0.22	0.25	0.175	0.199	0.135	0.156	0.424	0.345	0.152	0.193
B-splines	2	2	0.175	0.183	0.135	0.143	0.440	0.416	0.145	0.144
Partitioning	2	2	0.187	0.189	0.143	0.146	0.418	0.341	0.146	0.154
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.14	0.36	0.152	0.164	0.119	0.127	0.142	0.126	0.154	0.164
B-splines	3	1	0.165	0.163	0.134	0.129	0.184	0.161	0.145	0.144
Partitioning	3	1	0.170	0.163	0.133	0.129	0.137	0.161	0.147	0.144
Feasible Estimation										
Local Polynomial	0.19	0.26	0.146	0.177	0.114	0.137	0.141	0.152	0.152	0.188
B-splines	3	2	0.158	0.147	0.125	0.115	0.187	0.143	0.145	0.143
Partitioning	3	2	0.165	0.180	0.128	0.139	0.179	0.311	0.147	0.147

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.23: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.222	0.244	0.173	0.189	0.363	0.263	0.194	0.274
B-splines	8	5	0.218	0.374	0.171	0.290	0.322	0.974	0.177	0.241
Partitioning	8	5	0.261	0.288	0.204	0.222	0.321	0.277	0.183	0.251
Feasible Estimation										
Local Polynomial	0.11	0.18	0.255	0.260	0.189	0.198	0.723	0.612	0.179	0.278
B-splines	4	3	0.378	0.425	0.281	0.322	0.999	1.195	0.177	0.181
Partitioning	4	3	0.311	0.277	0.229	0.209	0.661	0.516	0.178	0.182
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.221	0.244	0.172	0.189	0.365	0.263	0.193	0.274
B-splines	8	5	0.217	0.374	0.170	0.290	0.322	0.974	0.177	0.239
Partitioning	8	5	0.261	0.288	0.203	0.222	0.320	0.277	0.183	0.251
Feasible Estimation										
Local Polynomial	0.12	0.18	0.277	0.261	0.198	0.199	0.847	0.618	0.179	0.275
B-splines	4	3	0.354	0.420	0.268	0.319	0.932	1.192	0.184	0.176
Partitioning	4	3	0.319	0.275	0.233	0.207	0.711	0.509	0.173	0.183
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.091	0.128	0.071	0.099	0.062	0.094	0.121	0.167
B-splines	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Partitioning	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Feasible Estimation										
Local Polynomial	0.24	0.26	0.131	0.176	0.099	0.133	0.103	0.136	0.178	0.206
B-splines	2	2	0.115	0.140	0.089	0.107	0.111	0.122	0.147	0.169
Partitioning	2	2	0.137	0.176	0.104	0.135	0.153	0.284	0.154	0.170
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.178	0.218	0.120	0.159	0.933	0.875	0.179	0.239
B-splines	5	5	0.197	0.215	0.140	0.157	0.936	0.911	0.177	0.192
Partitioning	5	5	0.232	0.300	0.173	0.230	0.897	0.766	0.178	0.251
Feasible Estimation										
Local Polynomial	0.23	0.26	0.179	0.212	0.120	0.153	0.933	0.898	0.179	0.209
B-splines	2	2	0.169	0.185	0.110	0.128	0.944	0.926	0.150	0.169
Partitioning	2	2	0.183	0.201	0.126	0.148	0.906	0.716	0.158	0.171
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.435	0.264	0.317	0.200	0.118	0.151	0.293	0.242
B-splines	5	5	0.378	0.305	0.311	0.238	0.236	0.205	0.228	0.250
Partitioning	5	5	0.298	0.297	0.228	0.227	0.148	0.218	0.259	0.251
Feasible Estimation										
Local Polynomial	0.09	0.21	0.266	0.270	0.199	0.203	0.153	0.151	0.252	0.243
B-splines	5	2	0.372	0.378	0.303	0.307	0.222	0.361	0.236	0.204
Partitioning	5	2	0.299	0.314	0.228	0.246	0.163	0.348	0.261	0.191
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.02	0.08	0.351	0.271	0.276	0.209	0.317	0.235	0.415	0.286
B-splines	22	6	0.380	0.313	0.280	0.228	0.301	0.192	0.561	0.489
Partitioning	22	6	0.470	0.321	0.359	0.247	0.431	0.416	0.629	0.276
Feasible Estimation										
Local Polynomial	0.05	0.14	0.568	0.280	0.367	0.217	0.203	0.183	1.509	0.270
B-splines	9	3	1.182	1.241	0.673	0.925	0.170	0.663	2.747	2.135
Partitioning	9	3	1.043	0.631	0.572	0.454	0.214	0.254	2.308	1.110
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.12	0.183	0.219	0.142	0.169	0.253	0.215	0.180	0.261
B-splines	5	4	0.202	0.199	0.159	0.157	0.423	0.331	0.178	0.187
Partitioning	5	4	0.217	0.254	0.169	0.195	0.309	0.378	0.178	0.223
Feasible Estimation										
Local Polynomial	0.21	0.25	0.181	0.202	0.140	0.157	0.414	0.331	0.181	0.215
B-splines	2	2	0.183	0.190	0.142	0.149	0.430	0.404	0.163	0.171
Partitioning	2	2	0.193	0.190	0.149	0.146	0.397	0.312	0.168	0.177
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.15	0.36	0.148	0.161	0.113	0.121	0.124	0.112	0.184	0.181
B-splines	3	1	0.155	0.158	0.122	0.123	0.147	0.138	0.167	0.174
Partitioning	3	1	0.167	0.158	0.130	0.123	0.122	0.138	0.167	0.174
Feasible Estimation										
Local Polynomial	0.19	0.26	0.145	0.176	0.111	0.133	0.128	0.137	0.180	0.207
B-splines	3	2	0.148	0.145	0.115	0.112	0.154	0.128	0.163	0.170
Partitioning	3	2	0.161	0.179	0.124	0.138	0.157	0.288	0.165	0.171

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.24: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 1$, $n = 1000$, $\sigma^2 = 4$, $X_i \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5)		(0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 1.1										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.230	0.249	0.177	0.186	0.325	0.234	0.300	0.401
B-splines	8	5	0.206	0.367	0.162	0.295	0.255	0.789	0.267	0.277
Partitioning	8	5	0.259	0.285	0.202	0.218	0.336	0.230	0.270	0.290
Feasible Estimation										
Local Polynomial	0.1	0.17	0.268	0.270	0.203	0.206	0.666	0.559	0.279	0.382
B-splines	5	3	0.379	0.457	0.298	0.364	0.875	1.096	0.247	0.276
Partitioning	5	3	0.280	0.248	0.213	0.190	0.492	0.330	0.260	0.262
Model 1.2										
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.229	0.249	0.176	0.186	0.326	0.234	0.300	0.401
B-splines	8	5	0.205	0.367	0.161	0.295	0.255	0.789	0.266	0.279
Partitioning	8	5	0.259	0.285	0.201	0.218	0.336	0.230	0.270	0.290
Feasible Estimation										
Local Polynomial	0.11	0.18	0.287	0.271	0.213	0.206	0.761	0.563	0.277	0.380
B-splines	4	3	0.323	0.455	0.252	0.363	0.682	1.095	0.261	0.275
Partitioning	4	3	0.267	0.247	0.201	0.189	0.424	0.328	0.239	0.262
Model 1.3										
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.092	0.129	0.071	0.096	0.064	0.088	0.140	0.241
B-splines	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Partitioning	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Feasible Estimation										
Local Polynomial	0.23	0.26	0.134	0.177	0.097	0.128	0.092	0.119	0.266	0.281
B-splines	2	2	0.117	0.141	0.090	0.105	0.107	0.111	0.186	0.254
Partitioning	2	2	0.139	0.177	0.106	0.134	0.145	0.264	0.197	0.259
Model 1.4										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.193	0.228	0.126	0.161	0.926	0.866	0.263	0.313
B-splines	5	5	0.209	0.226	0.148	0.163	0.908	0.885	0.242	0.261
Partitioning	5	5	0.241	0.304	0.180	0.233	0.851	0.678	0.250	0.290
Feasible Estimation										
Local Polynomial	0.22	0.25	0.195	0.224	0.127	0.156	0.923	0.887	0.267	0.284
B-splines	2	2	0.186	0.199	0.121	0.135	0.922	0.912	0.193	0.255
Partitioning	2	2	0.197	0.209	0.136	0.153	0.880	0.678	0.204	0.260
Model 1.5										
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.379	0.266	0.262	0.195	0.105	0.128	0.280	0.316
B-splines	5	5	0.490	0.280	0.361	0.205	0.166	0.129	0.401	0.268
Partitioning	5	5	0.428	0.309	0.309	0.238	0.144	0.219	0.348	0.295
Feasible Estimation										
Local Polynomial	0.1	0.19	0.269	0.267	0.193	0.195	0.126	0.131	0.317	0.337
B-splines	5	2	0.500	0.375	0.366	0.303	0.169	0.285	0.395	0.265
Partitioning	5	2	0.444	0.310	0.316	0.239	0.172	0.275	0.360	0.265
Model 1.6										
Infeasible Estimation										
Local Polynomial	0.03	0.09	0.319	0.259	0.240	0.192	0.244	0.193	0.567	0.413
B-splines	18	5	0.797	0.693	0.424	0.496	0.278	0.418	2.375	1.582
Partitioning	18	5	0.763	0.406	0.447	0.297	0.409	0.220	2.180	0.836
Feasible Estimation										
Local Polynomial	0.06	0.14	0.416	0.261	0.251	0.190	0.174	0.155	1.462	0.428
B-splines	9	3	1.493	1.091	0.777	0.708	0.184	0.444	1.746	1.716
Partitioning	9	3	1.370	0.699	0.664	0.481	0.260	0.191	2.002	1.529
Model 1.7										
Infeasible Estimation										
Local Polynomial	0.09	0.12	0.189	0.224	0.143	0.166	0.227	0.189	0.281	0.365
B-splines	6	4	0.172	0.197	0.134	0.154	0.192	0.269	0.253	0.263
Partitioning	6	4	0.221	0.255	0.172	0.195	0.280	0.383	0.260	0.273
Feasible Estimation										
Local Polynomial	0.2	0.24	0.192	0.207	0.148	0.158	0.391	0.303	0.270	0.294
B-splines	3	2	0.195	0.202	0.155	0.160	0.399	0.375	0.205	0.263
Partitioning	3	2	0.198	0.192	0.155	0.146	0.346	0.280	0.221	0.260
Model 1.8										
Infeasible Estimation										
Local Polynomial	0.17	0.37	0.144	0.158	0.106	0.114	0.106	0.101	0.275	0.264
B-splines	3	1	0.141	0.150	0.108	0.112	0.112	0.115	0.218	0.259
Partitioning	3	1	0.162	0.150	0.125	0.112	0.114	0.115	0.221	0.259
Feasible Estimation										
Local Polynomial	0.19	0.25	0.146	0.178	0.107	0.128	0.114	0.119	0.270	0.282
B-splines	3	2	0.137	0.144	0.104	0.107	0.129	0.115	0.207	0.257
Partitioning	3	2	0.156	0.180	0.120	0.136	0.150	0.267	0.212	0.261

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

C.2 Bivariate Simulations

C.2.1 Uniform Cell Boundaries

Table C.25: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.19	0.25	0.207	0.255	0.158	0.197	0.199	0.234	0.213	0.250	0.249	0.288
B-splines	9	4	0.214	0.233	0.167	0.180	0.230	0.271	0.156	0.233	0.222	0.242
Partitioning	9	4	0.243	0.286	0.187	0.217	0.213	1.024	0.202	0.564	0.209	0.242
Feasible Estimation												
Local Polynomial	0.3	0.27	0.207	0.217	0.155	0.170	0.088	0.161	0.122	0.145	0.112	0.134
B-splines	3	1	0.198	0.218	0.150	0.170	0.395	0.261	0.184	0.177	0.181	0.231
Partitioning	3	1	0.208	0.207	0.158	0.159	0.337	0.321	0.249	0.226	0.165	0.205
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.262	0.309	0.204	0.239	0.250	0.284	0.261	0.347	0.258	0.411
B-splines	9	4	0.382	0.233	0.314	0.179	0.481	0.207	0.388	0.229	0.245	0.243
Partitioning	9	4	0.311	0.294	0.246	0.224	0.275	1.131	0.246	0.576	0.221	0.244
Feasible Estimation												
Local Polynomial	0.33	0.24	0.490	0.494	0.392	0.391	0.706	0.757	0.607	0.615	0.287	0.297
B-splines	4	4	0.355	0.233	0.284	0.180	0.222	0.207	0.206	0.229	0.259	0.243
Partitioning	4	4	0.386	0.294	0.313	0.224	0.895	1.131	0.351	0.575	0.328	0.244
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.17	0.4	0.216	0.204	0.165	0.156	0.178	0.133	0.227	0.176	0.257	0.221
B-splines	9	4	0.223	0.224	0.175	0.171	0.151	0.207	0.170	0.229	0.223	0.242
Partitioning	9	4	0.259	0.285	0.202	0.217	0.174	1.047	0.207	0.566	0.215	0.242
Feasible Estimation												
Local Polynomial	0.34	0.28	0.264	0.276	0.170	0.185	0.118	0.137	0.247	0.259	0.198	0.212
B-splines	4	2	0.210	0.220	0.160	0.170	0.252	0.183	0.210	0.195	0.183	0.238
Partitioning	4	2	0.226	0.234	0.174	0.176	0.284	0.623	0.304	0.349	0.160	0.234
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.668	0.354	0.482	0.275	0.149	0.207	0.323	0.206	0.266	0.271
B-splines	9	9	0.731	0.387	0.579	0.304	0.240	0.307	0.209	0.243	0.282	0.270
Partitioning	9	9	0.665	0.455	0.524	0.349	0.196	0.366	0.265	0.346	0.249	0.304
Feasible Estimation												
Local Polynomial	0.21	0.28	0.638	0.540	0.524	0.443	0.405	0.412	0.668	0.379	0.601	0.907
B-splines	4	1	0.790	0.568	0.613	0.448	0.681	0.400	0.234	0.246	0.234	0.286
Partitioning	4	1	0.778	0.556	0.599	0.434	0.753	0.859	0.268	0.425	0.211	0.247
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.22	0.9	0.186	0.151	0.142	0.120	0.195	0.181	0.192	0.121	0.238	0.145
B-splines	4	1	0.140	0.209	0.108	0.164	0.247	0.200	0.195	0.170	0.181	0.234
Partitioning	4	1	0.160	0.178	0.125	0.142	0.303	0.188	0.225	0.146	0.158	0.188
Feasible Estimation												
Local Polynomial	0.26	0.29	0.122	0.122	0.096	0.093	0.206	0.104	0.105	0.128	0.105	0.128
B-splines	4	1	0.140	0.210	0.108	0.164	0.247	0.200	0.195	0.170	0.181	0.234
Partitioning	4	1	0.160	0.182	0.125	0.144	0.303	0.228	0.225	0.156	0.158	0.189
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.099	0.044	0.091	0.080	0.119	0.110	0.144
B-splines	1	1	0.089	0.178	0.068	0.136	0.044	0.114	0.077	0.164	0.129	0.230
Partitioning	1	1	0.076	0.140	0.060	0.109	0.044	0.092	0.076	0.140	0.097	0.185
Feasible Estimation												
Local Polynomial	0.4	0.31	0.461	0.468	0.405	0.407	0.062	0.110	0.112	0.139	0.647	0.658
B-splines	1	1	0.092	0.178	0.070	0.136	0.064	0.114	0.086	0.164	0.133	0.230
Partitioning	1	1	0.082	0.140	0.062	0.109	0.076	0.092	0.088	0.140	0.101	0.185
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.181	0.254	0.119	0.188	0.960	0.931	0.378	0.384	0.208	0.265
B-splines	9	9	0.216	0.293	0.153	0.219	0.962	0.947	0.382	0.404	0.222	0.251
Partitioning	9	9	0.261	0.436	0.194	0.330	0.954	0.924	0.396	0.465	0.205	0.298
Feasible Estimation												
Local Polynomial	0.34	0.28	0.148	0.173	0.079	0.110	0.981	0.983	0.369	0.374	0.104	0.128
B-splines	2	1	0.164	0.217	0.099	0.153	0.962	0.960	0.380	0.387	0.153	0.230
Partitioning	2	1	0.167	0.190	0.099	0.128	0.968	0.965	0.379	0.380	0.125	0.187
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.33	0.9	0.137	0.130	0.104	0.102	0.086	0.103	0.145	0.119	0.210	0.145
B-splines	1	1	0.104	0.179	0.079	0.137	0.050	0.116	0.078	0.164	0.130	0.230
Partitioning	1	1	0.094	0.142	0.072	0.110	0.050	0.105	0.078	0.143	0.097	0.186
Feasible Estimation												
Local Polynomial	0.34	0.28	0.097	0.132	0.073	0.101	0.062	0.102	0.104	0.128	0.104	0.128
B-splines	2	1	0.123	0.183	0.093	0.139	0.148	0.123	0.156	0.170	0.161	0.232
Partitioning	2	1	0.133	0.159	0.100	0.119	0.214	0.299	0.174	0.193	0.136	0.191

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.26: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.18	0.25	0.214	0.256	0.162	0.194	0.184	0.211	0.237	0.264	0.376	0.386
B-splines	9	4	0.226	0.236	0.177	0.178	0.221	0.243	0.176	0.258	0.330	0.399
Partitioning	9	4	0.246	0.287	0.192	0.218	0.192	0.841	0.219	0.589	0.277	0.372
Feasible Estimation												
Local Polynomial	0.29	0.27	0.224	0.231	0.173	0.184	0.072	0.145	0.134	0.159	0.135	0.150
B-splines	3	1	0.205	0.229	0.158	0.178	0.392	0.253	0.241	0.210	0.251	0.359
Partitioning	3	1	0.215	0.221	0.166	0.170	0.348	0.345	0.313	0.285	0.210	0.288
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.262	0.303	0.200	0.229	0.224	0.238	0.288	0.349	0.390	0.555
B-splines	9	4	0.376	0.233	0.308	0.175	0.440	0.186	0.364	0.254	0.403	0.401
Partitioning	9	4	0.310	0.294	0.245	0.223	0.263	0.946	0.257	0.605	0.340	0.370
Feasible Estimation												
Local Polynomial	0.3	0.24	0.508	0.513	0.413	0.414	0.671	0.701	0.589	0.586	0.316	0.335
B-splines	4	4	0.337	0.233	0.264	0.175	0.184	0.186	0.242	0.254	0.409	0.401
Partitioning	4	4	0.382	0.294	0.307	0.223	0.779	0.945	0.364	0.604	0.479	0.371
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.18	0.41	0.209	0.194	0.155	0.145	0.146	0.119	0.238	0.185	0.385	0.294
B-splines	9	4	0.217	0.225	0.167	0.168	0.133	0.186	0.191	0.254	0.336	0.399
Partitioning	9	4	0.255	0.285	0.197	0.216	0.143	0.835	0.228	0.593	0.288	0.376
Feasible Estimation												
Local Polynomial	0.34	0.28	0.217	0.233	0.141	0.155	0.114	0.126	0.233	0.246	0.188	0.204
B-splines	3	2	0.195	0.219	0.144	0.164	0.192	0.166	0.235	0.221	0.244	0.381
Partitioning	3	2	0.208	0.233	0.157	0.173	0.233	0.517	0.296	0.404	0.194	0.334
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.615	0.355	0.429	0.276	0.148	0.186	0.223	0.212	0.336	0.340
B-splines	9	9	0.681	0.392	0.511	0.307	0.155	0.276	0.212	0.253	0.435	0.489
Partitioning	9	9	0.632	0.458	0.479	0.352	0.166	0.289	0.259	0.378	0.346	0.475
Feasible Estimation												
Local Polynomial	0.22	0.28	0.604	0.516	0.485	0.411	0.362	0.365	0.547	0.344	0.734	0.953
B-splines	4	1	0.724	0.500	0.524	0.388	0.521	0.348	0.308	0.267	0.369	0.436
Partitioning	4	1	0.719	0.476	0.522	0.361	0.562	0.771	0.355	0.512	0.334	0.374
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.21	0.9	0.189	0.149	0.143	0.118	0.178	0.163	0.208	0.140	0.345	0.183
B-splines	4	1	0.142	0.208	0.109	0.160	0.233	0.176	0.214	0.197	0.254	0.366
Partitioning	4	1	0.163	0.178	0.127	0.140	0.277	0.170	0.223	0.167	0.204	0.269
Feasible Estimation												
Local Polynomial	0.26	0.29	0.127	0.125	0.102	0.094	0.198	0.095	0.126	0.146	0.126	0.146
B-splines	4	1	0.142	0.210	0.109	0.160	0.233	0.176	0.214	0.203	0.254	0.367
Partitioning	4	1	0.163	0.185	0.127	0.144	0.277	0.230	0.223	0.206	0.204	0.272
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.097	0.046	0.087	0.088	0.137	0.128	0.181
B-splines	1	1	0.090	0.180	0.069	0.134	0.045	0.102	0.084	0.183	0.159	0.359
Partitioning	1	1	0.078	0.142	0.061	0.109	0.045	0.088	0.085	0.153	0.108	0.263
Feasible Estimation												
Local Polynomial	0.4	0.31	0.422	0.430	0.363	0.367	0.058	0.094	0.128	0.153	0.662	0.662
B-splines	1	1	0.094	0.180	0.071	0.134	0.060	0.102	0.100	0.183	0.168	0.359
Partitioning	1	1	0.086	0.142	0.065	0.109	0.085	0.088	0.102	0.153	0.115	0.263
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.189	0.255	0.120	0.183	0.959	0.926	0.376	0.391	0.284	0.333
B-splines	9	9	0.225	0.298	0.157	0.218	0.961	0.938	0.383	0.405	0.331	0.448
Partitioning	9	9	0.267	0.440	0.199	0.335	0.943	0.898	0.404	0.476	0.275	0.465
Feasible Estimation												
Local Polynomial	0.34	0.28	0.159	0.184	0.084	0.113	0.982	0.986	0.367	0.380	0.122	0.146
B-splines	2	1	0.176	0.227	0.107	0.156	0.955	0.954	0.387	0.395	0.210	0.361
Partitioning	2	1	0.181	0.207	0.110	0.138	0.970	0.972	0.374	0.408	0.159	0.274
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.34	0.9	0.136	0.129	0.100	0.100	0.077	0.093	0.153	0.138	0.283	0.181
B-splines	1	1	0.102	0.180	0.078	0.134	0.052	0.104	0.087	0.183	0.159	0.358
Partitioning	1	1	0.092	0.144	0.071	0.110	0.052	0.095	0.088	0.154	0.110	0.269
Feasible Estimation												
Local Polynomial	0.34	0.28	0.094	0.132	0.071	0.099	0.060	0.091	0.121	0.146	0.122	0.146
B-splines	3	1	0.125	0.186	0.094	0.138	0.134	0.112	0.177	0.194	0.221	0.361
Partitioning	3	1	0.136	0.166	0.102	0.122	0.196	0.253	0.179	0.237	0.170	0.280

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.27: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE					
							(0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.17	0.24	0.232	0.249	0.159	0.179	0.151	0.161	0.336	0.335	1.295	1.400
B-splines	9	4	0.238	0.236	0.188	0.170	0.209	0.187	0.248	0.375	0.578	1.157
Partitioning	9	4	0.248	0.287	0.193	0.215	0.164	0.655	0.261	0.706	0.429	0.785
Feasible Estimation												
Local Polynomial	0.27	0.26	0.236	0.237	0.191	0.191	0.058	0.113	0.172	0.204	0.176	0.198
B-splines	4	2	0.207	0.239	0.162	0.182	0.332	0.223	0.353	0.340	0.369	0.939
Partitioning	4	2	0.218	0.247	0.170	0.189	0.377	0.421	0.390	0.467	0.261	0.596
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.259	0.291	0.186	0.206	0.180	0.169	0.379	0.425	4.109	7.040
B-splines	9	4	0.348	0.229	0.278	0.163	0.380	0.144	0.318	0.368	0.771	1.157
Partitioning	9	4	0.299	0.291	0.233	0.217	0.239	0.683	0.275	0.712	0.592	0.790
Feasible Estimation												
Local Polynomial	0.25	0.23	0.516	0.521	0.422	0.425	0.605	0.611	0.516	0.526	0.420	0.437
B-splines	4	4	0.295	0.229	0.221	0.163	0.200	0.144	0.372	0.368	0.711	1.157
Partitioning	4	4	0.346	0.291	0.269	0.217	0.621	0.683	0.440	0.712	0.689	0.791
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.19	0.45	0.189	0.170	0.131	0.122	0.097	0.093	0.295	0.230	0.817	0.440
B-splines	9	4	0.201	0.224	0.148	0.158	0.102	0.145	0.257	0.367	0.582	1.154
Partitioning	9	4	0.242	0.285	0.185	0.213	0.106	0.649	0.281	0.708	0.422	0.795
Feasible Estimation												
Local Polynomial	0.33	0.27	0.156	0.176	0.107	0.122	0.095	0.100	0.246	0.254	0.207	0.224
B-splines	3	3	0.165	0.215	0.118	0.152	0.146	0.128	0.278	0.339	0.345	1.003
Partitioning	3	3	0.176	0.243	0.130	0.174	0.190	0.484	0.293	0.544	0.239	0.686
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.501	0.342	0.326	0.262	0.137	0.150	0.286	0.263	0.588	0.600
B-splines	9	9	0.561	0.388	0.379	0.297	0.077	0.230	0.409	0.438	0.804	3.013
Partitioning	9	9	0.543	0.456	0.381	0.344	0.133	0.196	0.452	0.485	0.612	1.823
Feasible Estimation												
Local Polynomial	0.23	0.28	0.529	0.459	0.400	0.350	0.284	0.279	0.345	0.326	0.991	1.024
B-splines	4	2	0.574	0.431	0.363	0.338	0.278	0.282	0.505	0.397	0.612	1.216
Partitioning	4	2	0.578	0.391	0.375	0.300	0.335	0.720	0.515	0.715	0.509	0.835
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.21	0.9	0.188	0.141	0.136	0.109	0.147	0.122	0.276	0.196	0.688	0.262
B-splines	4	1	0.143	0.201	0.108	0.146	0.205	0.135	0.229	0.298	0.368	0.809
Partitioning	4	1	0.162	0.170	0.126	0.127	0.264	0.126	0.229	0.246	0.247	0.426
Feasible Estimation												
Local Polynomial	0.25	0.28	0.126	0.122	0.103	0.088	0.172	0.081	0.169	0.193	0.169	0.193
B-splines	4	1	0.143	0.206	0.108	0.149	0.205	0.133	0.229	0.312	0.368	0.879
Partitioning	4	1	0.162	0.195	0.126	0.142	0.264	0.260	0.229	0.348	0.247	0.529
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.081	0.122	0.062	0.091	0.043	0.074	0.097	0.172	0.136	0.244
B-splines	1	1	0.090	0.178	0.067	0.126	0.043	0.085	0.094	0.250	0.194	0.788
Partitioning	1	1	0.077	0.141	0.059	0.104	0.043	0.073	0.094	0.187	0.122	0.394
Feasible Estimation												
Local Polynomial	0.39	0.3	0.356	0.366	0.297	0.304	0.052	0.083	0.159	0.207	0.658	0.667
B-splines	1	1	0.098	0.179	0.072	0.126	0.067	0.086	0.127	0.250	0.246	0.789
Partitioning	1	1	0.093	0.143	0.068	0.104	0.086	0.088	0.124	0.191	0.156	0.398
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.199	0.250	0.120	0.171	0.951	0.914	0.393	0.414	0.413	0.587
B-splines	9	9	0.234	0.306	0.158	0.214	0.950	0.923	0.400	0.497	0.573	2.513
Partitioning	9	9	0.275	0.445	0.202	0.335	0.934	0.865	0.428	0.573	0.416	1.767
Feasible Estimation												
Local Polynomial	0.33	0.27	0.176	0.198	0.091	0.117	0.969	0.972	0.387	0.407	0.159	0.194
B-splines	3	2	0.195	0.246	0.121	0.165	0.930	0.934	0.394	0.454	0.337	0.913
Partitioning	3	2	0.203	0.247	0.131	0.166	0.916	0.919	0.373	0.519	0.230	0.568
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.36	0.9	0.125	0.124	0.089	0.093	0.060	0.078	0.175	0.172	0.378	0.244
B-splines	1	1	0.098	0.179	0.073	0.127	0.053	0.087	0.101	0.250	0.195	0.788
Partitioning	1	1	0.086	0.142	0.066	0.105	0.052	0.079	0.101	0.187	0.125	0.400
Feasible Estimation												
Local Polynomial	0.33	0.27	0.089	0.127	0.064	0.090	0.055	0.080	0.157	0.194	0.157	0.193
B-splines	3	2	0.127	0.195	0.092	0.137	0.128	0.105	0.206	0.291	0.332	0.911
Partitioning	3	2	0.139	0.200	0.104	0.139	0.179	0.344	0.201	0.408	0.224	0.561

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.28: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.23	0.29	0.345	0.530	0.264	0.405	0.281	0.383	0.363	0.501	0.462	0.668
B-splines	4	4	0.294	0.451	0.228	0.346	0.578	0.446	0.399	0.459	0.362	0.485
Partitioning	4	4	0.335	0.567	0.260	0.431	0.600	2.040	0.484	1.127	0.322	0.483
Feasible Estimation												
Local Polynomial	0.3	0.26	0.253	0.304	0.196	0.240	0.142	0.244	0.219	0.270	0.214	0.265
B-splines	3	2	0.292	0.409	0.225	0.313	0.522	0.379	0.363	0.392	0.348	0.471
Partitioning	3	2	0.321	0.431	0.247	0.316	0.543	1.273	0.436	0.718	0.306	0.420
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.437	0.549	0.337	0.423	0.382	0.479	0.463	0.566	0.513	0.673
B-splines	9	4	0.492	0.451	0.396	0.346	0.514	0.414	0.469	0.457	0.455	0.484
Partitioning	9	4	0.509	0.571	0.397	0.435	0.396	2.085	0.426	1.135	0.418	0.484
Feasible Estimation												
Local Polynomial	0.31	0.25	0.510	0.538	0.408	0.426	0.721	0.783	0.632	0.658	0.340	0.369
B-splines	4	4	0.425	0.455	0.337	0.350	0.413	0.426	0.401	0.451	0.404	0.483
Partitioning	4	4	0.471	0.564	0.375	0.431	1.006	1.961	0.536	1.094	0.426	0.483
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.21	0.46	0.363	0.360	0.275	0.275	0.271	0.220	0.390	0.304	0.481	0.387
B-splines	4	4	0.311	0.447	0.239	0.342	0.434	0.414	0.392	0.457	0.362	0.484
Partitioning	4	4	0.351	0.567	0.273	0.431	0.544	2.057	0.495	1.127	0.318	0.483
Feasible Estimation												
Local Polynomial	0.31	0.27	0.300	0.348	0.206	0.251	0.161	0.227	0.306	0.343	0.268	0.310
B-splines	3	2	0.307	0.411	0.233	0.313	0.401	0.335	0.391	0.402	0.349	0.474
Partitioning	3	2	0.339	0.450	0.261	0.328	0.509	1.403	0.480	0.772	0.300	0.440
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.707	0.529	0.523	0.415	0.205	0.336	0.410	0.406	0.449	0.533
B-splines	9	9	0.793	0.604	0.630	0.471	0.299	0.433	0.339	0.444	0.479	0.512
Partitioning	9	9	0.777	0.863	0.619	0.657	0.355	0.731	0.440	0.687	0.435	0.599
Feasible Estimation												
Local Polynomial	0.23	0.27	0.685	0.581	0.554	0.469	0.432	0.448	0.671	0.440	0.649	0.939
B-splines	4	2	0.830	0.661	0.640	0.524	0.793	0.493	0.413	0.430	0.390	0.500
Partitioning	4	2	0.835	0.672	0.649	0.532	0.927	1.563	0.472	0.826	0.344	0.441
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.27	0.9	0.314	0.265	0.241	0.209	0.297	0.241	0.323	0.239	0.433	0.288
B-splines	4	1	0.270	0.373	0.207	0.287	0.423	0.281	0.389	0.333	0.361	0.462
Partitioning	4	1	0.312	0.302	0.243	0.235	0.548	0.246	0.449	0.284	0.316	0.370
Feasible Estimation												
Local Polynomial	0.27	0.27	0.195	0.244	0.152	0.188	0.258	0.210	0.209	0.260	0.209	0.260
B-splines	4	2	0.277	0.398	0.212	0.303	0.436	0.321	0.394	0.383	0.369	0.468
Partitioning	4	2	0.317	0.403	0.247	0.294	0.553	1.136	0.452	0.661	0.325	0.407
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.166	0.252	0.129	0.198	0.088	0.183	0.161	0.238	0.221	0.288
B-splines	1	1	0.177	0.357	0.136	0.273	0.087	0.228	0.153	0.329	0.258	0.460
Partitioning	1	1	0.153	0.281	0.119	0.217	0.087	0.183	0.153	0.280	0.193	0.369
Feasible Estimation												
Local Polynomial	0.34	0.28	0.482	0.513	0.416	0.433	0.126	0.213	0.213	0.263	0.671	0.697
B-splines	2	1	0.226	0.365	0.169	0.278	0.270	0.239	0.299	0.340	0.319	0.463
Partitioning	2	1	0.241	0.315	0.176	0.234	0.377	0.582	0.334	0.374	0.262	0.379
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.296	0.468	0.217	0.358	0.970	0.970	0.456	0.519	0.416	0.531
B-splines	9	9	0.377	0.549	0.287	0.420	0.978	0.998	0.470	0.553	0.444	0.502
Partitioning	9	9	0.480	0.853	0.370	0.648	1.007	1.130	0.532	0.757	0.410	0.596
Feasible Estimation												
Local Polynomial	0.31	0.27	0.207	0.274	0.140	0.201	0.985	0.997	0.411	0.435	0.208	0.261
B-splines	3	2	0.273	0.405	0.198	0.304	1.001	0.982	0.486	0.521	0.338	0.466
Partitioning	3	2	0.298	0.405	0.216	0.290	1.032	1.374	0.523	0.733	0.289	0.407
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.42	0.9	0.239	0.254	0.180	0.199	0.129	0.189	0.237	0.238	0.363	0.288
B-splines	1	1	0.185	0.357	0.142	0.273	0.091	0.229	0.154	0.329	0.259	0.460
Partitioning	1	1	0.162	0.282	0.126	0.218	0.091	0.190	0.154	0.282	0.193	0.370
Feasible Estimation												
Local Polynomial	0.31	0.27	0.173	0.249	0.132	0.192	0.129	0.210	0.209	0.261	0.208	0.261
B-splines	3	2	0.251	0.392	0.190	0.297	0.346	0.309	0.349	0.387	0.339	0.468
Partitioning	3	2	0.282	0.408	0.213	0.292	0.472	1.230	0.397	0.699	0.292	0.413

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.29: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.22	0.28	0.358	0.540	0.268	0.400	0.264	0.350	0.404	0.528	0.676	0.883
B-splines	4	4	0.304	0.455	0.235	0.340	0.526	0.402	0.451	0.510	0.507	0.798
Partitioning	4	4	0.342	0.568	0.266	0.431	0.558	1.666	0.514	1.175	0.411	0.742
Feasible Estimation												
Local Polynomial	0.3	0.26	0.269	0.315	0.211	0.248	0.129	0.222	0.253	0.300	0.257	0.296
B-splines	3	2	0.304	0.415	0.235	0.313	0.489	0.345	0.424	0.431	0.493	0.747
Partitioning	3	2	0.334	0.437	0.259	0.322	0.516	1.036	0.484	0.779	0.393	0.618
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.436	0.541	0.329	0.408	0.345	0.408	0.501	0.582	0.762	0.854
B-splines	9	4	0.489	0.453	0.391	0.338	0.466	0.370	0.464	0.508	0.710	0.800
Partitioning	9	4	0.509	0.572	0.398	0.434	0.356	1.726	0.457	1.185	0.592	0.738
Feasible Estimation												
Local Polynomial	0.3	0.25	0.529	0.556	0.429	0.447	0.677	0.717	0.633	0.639	0.376	0.418
B-splines	4	4	0.411	0.457	0.321	0.342	0.361	0.379	0.447	0.493	0.608	0.795
Partitioning	4	4	0.469	0.565	0.371	0.430	0.878	1.655	0.530	1.122	0.600	0.750
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.22	0.47	0.351	0.342	0.259	0.256	0.228	0.201	0.404	0.325	0.684	0.517
B-splines	4	1	0.305	0.377	0.231	0.281	0.373	0.231	0.432	0.374	0.509	0.717
Partitioning	4	1	0.345	0.311	0.267	0.236	0.488	0.186	0.485	0.338	0.408	0.545
Feasible Estimation												
Local Polynomial	0.32	0.27	0.262	0.317	0.181	0.227	0.157	0.207	0.312	0.355	0.281	0.328
B-splines	3	2	0.297	0.415	0.223	0.307	0.336	0.306	0.419	0.441	0.488	0.754
Partitioning	3	2	0.328	0.455	0.250	0.330	0.449	1.088	0.467	0.844	0.388	0.641
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.659	0.523	0.472	0.404	0.197	0.299	0.347	0.420	0.596	0.671
B-splines	9	9	0.749	0.608	0.570	0.466	0.221	0.382	0.357	0.475	0.722	0.918
Partitioning	9	9	0.750	0.866	0.584	0.662	0.293	0.578	0.457	0.747	0.592	0.936
Feasible Estimation												
Local Polynomial	0.23	0.27	0.652	0.560	0.517	0.444	0.392	0.401	0.545	0.428	0.814	0.987
B-splines	4	2	0.765	0.636	0.555	0.492	0.620	0.440	0.480	0.483	0.574	0.798
Partitioning	4	2	0.775	0.650	0.576	0.502	0.703	1.290	0.525	0.934	0.491	0.687
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.27	0.9	0.320	0.264	0.241	0.206	0.273	0.224	0.353	0.275	0.612	0.361
B-splines	4	1	0.274	0.374	0.209	0.281	0.386	0.250	0.428	0.374	0.508	0.719
Partitioning	4	1	0.316	0.304	0.246	0.234	0.498	0.231	0.446	0.314	0.408	0.527
Feasible Estimation												
Local Polynomial	0.27	0.27	0.204	0.249	0.161	0.187	0.252	0.187	0.251	0.294	0.251	0.294
B-splines	4	2	0.279	0.401	0.212	0.298	0.393	0.286	0.433	0.425	0.510	0.741
Partitioning	4	2	0.320	0.406	0.249	0.295	0.501	0.863	0.451	0.682	0.413	0.602
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.166	0.251	0.128	0.194	0.092	0.174	0.177	0.274	0.256	0.361
B-splines	1	1	0.180	0.359	0.137	0.267	0.090	0.204	0.169	0.367	0.318	0.717
Partitioning	1	1	0.156	0.285	0.122	0.217	0.090	0.176	0.169	0.307	0.217	0.526
Feasible Estimation												
Local Polynomial	0.35	0.28	0.446	0.481	0.377	0.399	0.118	0.185	0.249	0.298	0.701	0.711
B-splines	2	1	0.233	0.370	0.172	0.274	0.250	0.221	0.333	0.388	0.426	0.724
Partitioning	2	1	0.249	0.328	0.183	0.241	0.356	0.497	0.342	0.471	0.326	0.550
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.301	0.461	0.216	0.344	0.969	0.956	0.461	0.537	0.568	0.667
B-splines	9	9	0.385	0.553	0.290	0.413	0.976	0.974	0.479	0.571	0.663	0.896
Partitioning	9	9	0.485	0.857	0.376	0.654	0.974	1.031	0.556	0.796	0.550	0.929
Feasible Estimation												
Local Polynomial	0.32	0.27	0.217	0.283	0.145	0.203	0.990	1.003	0.420	0.458	0.247	0.293
B-splines	3	2	0.286	0.415	0.206	0.306	0.973	0.970	0.520	0.552	0.479	0.742
Partitioning	3	2	0.313	0.426	0.229	0.305	1.038	1.290	0.515	0.800	0.377	0.608
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.42	0.9	0.235	0.253	0.174	0.196	0.121	0.177	0.251	0.275	0.480	0.361
B-splines	1	1	0.187	0.359	0.142	0.267	0.094	0.204	0.170	0.367	0.318	0.717
Partitioning	1	1	0.163	0.286	0.127	0.218	0.094	0.179	0.171	0.307	0.217	0.530
Feasible Estimation												
Local Polynomial	0.32	0.27	0.175	0.252	0.132	0.190	0.121	0.186	0.247	0.293	0.247	0.293
B-splines	3	2	0.258	0.397	0.194	0.293	0.311	0.270	0.391	0.433	0.484	0.744
Partitioning	3	2	0.290	0.417	0.220	0.298	0.434	0.975	0.408	0.770	0.378	0.614

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.30: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.21	0.27	0.366	0.453	0.261	0.324	0.220	0.268	0.554	0.587	1.384	1.542
B-splines	4	4	0.309	0.453	0.236	0.322	0.427	0.313	0.535	0.738	0.738	2.308
Partitioning	4	4	0.344	0.568	0.268	0.425	0.538	1.285	0.558	1.407	0.500	1.558
Feasible Estimation												
Local Polynomial	0.29	0.26	0.279	0.320	0.223	0.247	0.112	0.183	0.332	0.398	0.334	0.392
B-splines	4	3	0.310	0.434	0.237	0.313	0.413	0.305	0.518	0.673	0.724	2.065
Partitioning	4	3	0.341	0.496	0.265	0.362	0.522	1.012	0.536	1.115	0.496	1.364
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.423	0.511	0.303	0.365	0.277	0.290	0.654	0.702	6.256	5.066
B-splines	9	4	0.464	0.449	0.360	0.319	0.403	0.288	0.473	0.734	1.265	2.308
Partitioning	9	4	0.499	0.570	0.385	0.426	0.297	1.295	0.524	1.409	0.933	1.552
Feasible Estimation												
Local Polynomial	0.27	0.24	0.536	0.563	0.436	0.453	0.612	0.628	0.589	0.617	0.506	0.558
B-splines	4	4	0.371	0.450	0.278	0.320	0.312	0.294	0.574	0.723	0.930	2.241
Partitioning	4	4	0.442	0.566	0.341	0.423	0.754	1.266	0.619	1.378	0.809	1.549
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.25	0.51	0.317	0.304	0.221	0.219	0.156	0.161	0.485	0.415	1.092	0.750
B-splines	4	1	0.288	0.368	0.212	0.261	0.304	0.185	0.484	0.520	0.740	1.574
Partitioning	4	1	0.327	0.298	0.252	0.218	0.426	0.157	0.501	0.415	0.502	0.791
Feasible Estimation												
Local Polynomial	0.31	0.26	0.214	0.277	0.151	0.197	0.132	0.173	0.373	0.424	0.348	0.406
B-splines	3	3	0.281	0.424	0.205	0.299	0.286	0.255	0.470	0.684	0.709	2.067
Partitioning	3	3	0.312	0.497	0.236	0.356	0.394	1.025	0.482	1.163	0.480	1.395
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.549	0.492	0.370	0.368	0.169	0.229	0.430	0.520	0.931	1.182
B-splines	9	9	0.640	0.606	0.450	0.443	0.148	0.306	0.537	0.785	1.290	5.271
Partitioning	9	9	0.674	0.865	0.502	0.649	0.220	0.392	0.634	0.959	0.950	3.574
Feasible Estimation												
Local Polynomial	0.25	0.27	0.573	0.506	0.431	0.387	0.303	0.311	0.398	0.475	1.125	1.075
B-splines	4	3	0.621	0.581	0.410	0.436	0.384	0.352	0.635	0.737	0.874	2.262
Partitioning	4	3	0.638	0.615	0.443	0.463	0.505	1.205	0.640	1.262	0.656	1.540
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.26	0.9	0.314	0.254	0.228	0.192	0.225	0.175	0.454	0.357	1.003	0.498
B-splines	4	1	0.273	0.369	0.203	0.263	0.330	0.199	0.457	0.527	0.736	1.579
Partitioning	4	1	0.314	0.298	0.244	0.220	0.456	0.177	0.456	0.409	0.494	0.805
Feasible Estimation												
Local Polynomial	0.27	0.26	0.201	0.248	0.156	0.177	0.214	0.163	0.333	0.390	0.333	0.390
B-splines	4	2	0.275	0.414	0.205	0.292	0.333	0.245	0.461	0.649	0.737	2.018
Partitioning	4	2	0.315	0.463	0.245	0.328	0.457	0.917	0.459	1.004	0.498	1.308
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.161	0.243	0.123	0.183	0.087	0.147	0.193	0.343	0.273	0.487
B-splines	1	1	0.179	0.357	0.133	0.252	0.087	0.171	0.189	0.499	0.389	1.577
Partitioning	1	1	0.153	0.283	0.118	0.207	0.086	0.147	0.189	0.374	0.244	0.789
Feasible Estimation												
Local Polynomial	0.33	0.27	0.384	0.424	0.316	0.341	0.105	0.163	0.324	0.395	0.710	0.741
B-splines	3	2	0.247	0.388	0.179	0.272	0.245	0.208	0.408	0.578	0.667	1.830
Partitioning	3	2	0.270	0.398	0.200	0.276	0.348	0.686	0.396	0.788	0.443	1.134
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.300	0.432	0.206	0.308	0.954	0.929	0.512	0.613	0.825	1.174
B-splines	9	9	0.387	0.557	0.282	0.396	0.956	0.943	0.536	0.816	1.146	5.026
Partitioning	9	9	0.485	0.859	0.372	0.646	0.949	0.929	0.624	1.011	0.831	3.534
Feasible Estimation												
Local Polynomial	0.31	0.26	0.228	0.292	0.147	0.200	0.972	0.982	0.483	0.534	0.324	0.390
B-splines	3	3	0.300	0.443	0.213	0.313	0.949	0.945	0.553	0.748	0.705	2.066
Partitioning	3	3	0.328	0.499	0.242	0.356	0.959	1.242	0.532	1.147	0.478	1.355
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.45	0.9	0.216	0.244	0.156	0.184	0.100	0.150	0.280	0.343	0.573	0.488
B-splines	1	1	0.183	0.357	0.136	0.253	0.092	0.171	0.191	0.499	0.388	1.576
Partitioning	1	1	0.158	0.283	0.122	0.208	0.091	0.150	0.191	0.374	0.245	0.792
Feasible Estimation												
Local Polynomial	0.31	0.26	0.171	0.250	0.123	0.178	0.108	0.163	0.323	0.390	0.323	0.389
B-splines	3	3	0.261	0.416	0.191	0.292	0.276	0.245	0.438	0.655	0.705	2.048
Partitioning	3	3	0.294	0.477	0.224	0.338	0.397	0.973	0.431	1.068	0.474	1.340

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.31: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.161	0.191	0.124	0.148	0.167	0.184	0.157	0.193	0.177	0.220
B-splines	9	4	0.175	0.173	0.135	0.134	0.215	0.231	0.115	0.167	0.160	0.166
Partitioning	9	4	0.180	0.206	0.139	0.156	0.181	0.654	0.140	0.385	0.151	0.169
Feasible Estimation												
Local Polynomial	0.28	0.27	0.199	0.201	0.146	0.155	0.084	0.149	0.096	0.113	0.084	0.096
B-splines	4	1	0.166	0.177	0.124	0.138	0.419	0.243	0.167	0.125	0.141	0.162
Partitioning	4	1	0.177	0.175	0.133	0.134	0.328	0.235	0.248	0.176	0.138	0.159
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.206	0.233	0.161	0.180	0.202	0.213	0.196	0.273	0.182	0.323
B-splines	16	4	0.210	0.173	0.166	0.133	0.408	0.148	0.267	0.157	0.175	0.168
Partitioning	16	4	0.253	0.218	0.196	0.166	0.555	0.830	0.377	0.396	0.167	0.172
Feasible Estimation												
Local Polynomial	0.32	0.23	0.487	0.489	0.390	0.387	0.703	0.747	0.602	0.604	0.277	0.285
B-splines	4	4	0.343	0.173	0.275	0.133	0.169	0.148	0.149	0.157	0.228	0.168
Partitioning	4	4	0.373	0.218	0.305	0.166	0.884	0.830	0.290	0.396	0.311	0.172
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.15	0.37	0.168	0.156	0.129	0.120	0.141	0.106	0.168	0.133	0.186	0.174
B-splines	16	4	0.177	0.160	0.138	0.122	0.279	0.148	0.223	0.157	0.174	0.167
Partitioning	16	4	0.233	0.205	0.181	0.156	0.425	0.667	0.322	0.386	0.177	0.171
Feasible Estimation												
Local Polynomial	0.34	0.28	0.259	0.265	0.164	0.172	0.114	0.121	0.232	0.240	0.179	0.189
B-splines	4	3	0.187	0.167	0.145	0.129	0.222	0.148	0.145	0.146	0.140	0.165
Partitioning	4	3	0.200	0.196	0.156	0.149	0.213	0.533	0.248	0.320	0.127	0.180
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.666	0.318	0.479	0.242	0.140	0.175	0.306	0.151	0.197	0.197
B-splines	9	9	0.726	0.342	0.575	0.269	0.223	0.276	0.168	0.179	0.194	0.190
Partitioning	9	9	0.652	0.348	0.512	0.267	0.148	0.233	0.203	0.239	0.175	0.213
Feasible Estimation												
Local Polynomial	0.19	0.27	0.610	0.530	0.509	0.438	0.394	0.394	0.667	0.363	0.588	0.902
B-splines	4	1	0.733	0.410	0.579	0.322	0.303	0.392	0.166	0.189	0.192	0.186
Partitioning	4	1	0.667	0.359	0.521	0.274	0.277	0.757	0.200	0.387	0.173	0.175
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.145	0.123	0.112	0.099	0.156	0.170	0.143	0.085	0.174	0.108
B-splines	4	1	0.105	0.169	0.081	0.135	0.204	0.186	0.140	0.119	0.136	0.166
Partitioning	4	1	0.118	0.150	0.092	0.122	0.244	0.179	0.152	0.103	0.122	0.145
Feasible Estimation												
Local Polynomial	0.23	0.29	0.097	0.088	0.077	0.068	0.173	0.078	0.075	0.089	0.075	0.089
B-splines	4	1	0.105	0.169	0.081	0.134	0.204	0.185	0.140	0.119	0.136	0.166
Partitioning	4	1	0.118	0.153	0.092	0.123	0.244	0.204	0.152	0.119	0.122	0.146
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.060	0.090	0.046	0.071	0.033	0.066	0.057	0.084	0.083	0.107
B-splines	1	1	0.063	0.127	0.048	0.097	0.033	0.081	0.055	0.117	0.094	0.162
Partitioning	1	1	0.055	0.101	0.043	0.078	0.033	0.066	0.055	0.099	0.072	0.142
Feasible Estimation												
Local Polynomial	0.41	0.31	0.459	0.462	0.404	0.405	0.045	0.075	0.078	0.096	0.646	0.652
B-splines	1	1	0.066	0.127	0.050	0.097	0.044	0.081	0.066	0.117	0.097	0.162
Partitioning	1	1	0.060	0.101	0.045	0.078	0.053	0.066	0.066	0.099	0.074	0.142
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.155	0.198	0.091	0.139	0.960	0.924	0.356	0.353	0.151	0.188
B-splines	9	9	0.176	0.224	0.115	0.162	0.963	0.938	0.359	0.360	0.160	0.171
Partitioning	9	9	0.204	0.319	0.144	0.239	0.937	0.883	0.356	0.384	0.148	0.199
Feasible Estimation												
Local Polynomial	0.34	0.28	0.137	0.152	0.064	0.085	0.984	0.986	0.360	0.362	0.074	0.090
B-splines	2	1	0.145	0.177	0.077	0.115	0.967	0.959	0.364	0.359	0.114	0.162
Partitioning	2	1	0.147	0.161	0.077	0.099	0.966	0.959	0.361	0.356	0.095	0.144
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.29	0.9	0.104	0.096	0.079	0.075	0.071	0.080	0.111	0.086	0.159	0.108
B-splines	4	1	0.097	0.128	0.075	0.098	0.143	0.084	0.139	0.117	0.135	0.162
Partitioning	4	1	0.114	0.103	0.089	0.080	0.205	0.082	0.151	0.101	0.122	0.145
Feasible Estimation												
Local Polynomial	0.34	0.28	0.079	0.102	0.059	0.078	0.047	0.072	0.073	0.090	0.073	0.090
B-splines	3	1	0.093	0.130	0.070	0.099	0.108	0.088	0.112	0.118	0.121	0.162
Partitioning	3	1	0.102	0.113	0.077	0.085	0.164	0.168	0.122	0.129	0.106	0.146

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.32: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.16	0.23	0.166	0.190	0.126	0.144	0.150	0.161	0.178	0.196	0.253	0.268
B-splines	9	4	0.186	0.174	0.146	0.133	0.209	0.205	0.131	0.185	0.210	0.240
Partitioning	9	4	0.183	0.205	0.142	0.156	0.164	0.573	0.152	0.393	0.184	0.232
Feasible Estimation												
Local Polynomial	0.27	0.27	0.215	0.213	0.165	0.170	0.059	0.126	0.101	0.115	0.098	0.105
B-splines	4	1	0.172	0.187	0.132	0.146	0.393	0.236	0.209	0.153	0.169	0.227
Partitioning	4	1	0.181	0.184	0.138	0.143	0.341	0.258	0.288	0.200	0.155	0.199
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.203	0.227	0.156	0.171	0.177	0.174	0.210	0.262	0.259	0.401
B-splines	16	4	0.205	0.170	0.159	0.128	0.353	0.128	0.289	0.182	0.246	0.242
Partitioning	16	4	0.251	0.216	0.195	0.164	0.510	0.696	0.411	0.404	0.217	0.233
Feasible Estimation												
Local Polynomial	0.29	0.23	0.505	0.508	0.411	0.410	0.675	0.704	0.571	0.572	0.313	0.320
B-splines	4	4	0.327	0.170	0.257	0.128	0.165	0.128	0.176	0.182	0.334	0.242
Partitioning	4	4	0.362	0.216	0.292	0.164	0.752	0.696	0.317	0.404	0.429	0.233
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.16	0.39	0.160	0.148	0.120	0.110	0.109	0.091	0.177	0.138	0.258	0.203
B-splines	9	4	0.176	0.159	0.137	0.118	0.117	0.129	0.148	0.182	0.212	0.240
Partitioning	9	4	0.194	0.203	0.150	0.154	0.101	0.582	0.161	0.397	0.192	0.235
Feasible Estimation												
Local Polynomial	0.34	0.28	0.210	0.217	0.133	0.140	0.104	0.103	0.214	0.214	0.164	0.166
B-splines	4	3	0.172	0.164	0.128	0.124	0.172	0.130	0.166	0.171	0.169	0.236
Partitioning	4	3	0.182	0.190	0.137	0.142	0.179	0.459	0.239	0.328	0.137	0.230
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.613	0.322	0.425	0.246	0.138	0.158	0.190	0.155	0.223	0.224
B-splines	9	9	0.677	0.346	0.507	0.274	0.136	0.254	0.154	0.192	0.297	0.275
Partitioning	9	9	0.618	0.349	0.464	0.269	0.133	0.188	0.187	0.255	0.235	0.278
Feasible Estimation												
Local Polynomial	0.19	0.27	0.580	0.506	0.471	0.403	0.345	0.344	0.571	0.325	0.690	0.945
B-splines	4	2	0.688	0.399	0.510	0.323	0.267	0.346	0.185	0.206	0.290	0.257
Partitioning	4	2	0.642	0.335	0.475	0.261	0.285	0.707	0.213	0.412	0.239	0.238
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.145	0.120	0.110	0.097	0.139	0.149	0.158	0.098	0.235	0.125
B-splines	4	1	0.105	0.166	0.081	0.130	0.193	0.162	0.151	0.146	0.168	0.235
Partitioning	4	1	0.118	0.147	0.092	0.118	0.225	0.156	0.154	0.126	0.136	0.182
Feasible Estimation												
Local Polynomial	0.23	0.29	0.099	0.087	0.079	0.066	0.161	0.067	0.088	0.096	0.088	0.096
B-splines	4	1	0.105	0.166	0.081	0.130	0.193	0.162	0.151	0.149	0.168	0.234
Partitioning	4	1	0.118	0.151	0.092	0.120	0.225	0.187	0.154	0.139	0.136	0.186
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.059	0.089	0.046	0.069	0.033	0.061	0.060	0.094	0.089	0.122
B-splines	1	1	0.064	0.127	0.049	0.094	0.033	0.072	0.058	0.129	0.109	0.226
Partitioning	1	1	0.055	0.100	0.043	0.076	0.033	0.060	0.057	0.107	0.077	0.171
Feasible Estimation												
Local Polynomial	0.4	0.31	0.419	0.423	0.362	0.364	0.042	0.065	0.086	0.103	0.646	0.651
B-splines	1	1	0.066	0.127	0.050	0.094	0.043	0.072	0.067	0.129	0.115	0.226
Partitioning	1	1	0.060	0.100	0.046	0.076	0.054	0.060	0.068	0.107	0.083	0.171
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.164	0.202	0.093	0.137	0.957	0.918	0.359	0.355	0.183	0.215
B-splines	9	9	0.185	0.230	0.118	0.161	0.960	0.930	0.364	0.371	0.209	0.254
Partitioning	9	9	0.211	0.323	0.148	0.243	0.937	0.858	0.369	0.390	0.179	0.267
Feasible Estimation												
Local Polynomial	0.34	0.28	0.149	0.162	0.068	0.087	0.978	0.979	0.361	0.363	0.084	0.097
B-splines	2	1	0.157	0.186	0.085	0.118	0.954	0.952	0.368	0.369	0.145	0.226
Partitioning	2	1	0.160	0.174	0.087	0.105	0.956	0.946	0.357	0.369	0.112	0.179
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.3	0.9	0.103	0.094	0.076	0.072	0.061	0.070	0.119	0.094	0.197	0.122
B-splines	4	1	0.096	0.127	0.073	0.095	0.125	0.074	0.151	0.129	0.168	0.226
Partitioning	4	1	0.113	0.102	0.088	0.078	0.177	0.072	0.154	0.109	0.136	0.176
Feasible Estimation												
Local Polynomial	0.34	0.28	0.074	0.097	0.055	0.073	0.043	0.063	0.083	0.096	0.083	0.097
B-splines	3	1	0.091	0.131	0.069	0.097	0.099	0.079	0.126	0.138	0.150	0.225
Partitioning	3	1	0.101	0.118	0.076	0.086	0.145	0.178	0.131	0.156	0.119	0.183

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.33: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.15	0.22	0.174	0.187	0.125	0.134	0.127	0.130	0.237	0.229	0.739	0.554
B-splines	9	4	0.203	0.175	0.162	0.128	0.202	0.159	0.192	0.236	0.391	0.633
Partitioning	9	4	0.188	0.206	0.147	0.154	0.150	0.456	0.176	0.464	0.305	0.496
Feasible Estimation												
Local Polynomial	0.25	0.26	0.227	0.221	0.183	0.180	0.043	0.098	0.124	0.144	0.136	0.145
B-splines	4	2	0.180	0.194	0.140	0.150	0.326	0.212	0.318	0.241	0.256	0.559
Partitioning	4	2	0.186	0.199	0.144	0.154	0.350	0.322	0.379	0.324	0.194	0.389
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.201	0.216	0.145	0.154	0.146	0.132	0.263	0.293	0.637	1.093
B-splines	16	4	0.195	0.166	0.145	0.118	0.297	0.103	0.315	0.231	0.539	0.632
Partitioning	16	4	0.249	0.212	0.190	0.158	0.426	0.498	0.459	0.481	0.438	0.509
Feasible Estimation												
Local Polynomial	0.22	0.22	0.513	0.516	0.420	0.422	0.606	0.613	0.510	0.514	0.392	0.400
B-splines	8	4	0.323	0.166	0.257	0.118	0.355	0.103	0.271	0.231	0.567	0.632
Partitioning	8	4	0.267	0.212	0.209	0.158	0.313	0.498	0.229	0.481	0.515	0.509
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.17	0.42	0.147	0.130	0.102	0.092	0.079	0.073	0.211	0.162	0.537	0.344
B-splines	9	4	0.158	0.158	0.116	0.111	0.089	0.104	0.205	0.230	0.401	0.633
Partitioning	9	4	0.180	0.202	0.136	0.150	0.078	0.444	0.198	0.472	0.295	0.506
Feasible Estimation												
Local Polynomial	0.34	0.28	0.146	0.155	0.098	0.104	0.089	0.083	0.207	0.197	0.164	0.164
B-splines	3	3	0.140	0.157	0.098	0.111	0.114	0.102	0.228	0.222	0.243	0.597
Partitioning	3	3	0.147	0.184	0.107	0.134	0.147	0.365	0.245	0.399	0.186	0.462
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.496	0.313	0.320	0.239	0.131	0.133	0.227	0.172	0.456	0.380
B-splines	9	9	0.555	0.344	0.370	0.269	0.054	0.217	0.360	0.249	0.593	1.193
Partitioning	9	9	0.527	0.348	0.359	0.262	0.113	0.137	0.375	0.295	0.486	0.791
Feasible Estimation												
Local Polynomial	0.21	0.27	0.512	0.450	0.390	0.342	0.273	0.263	0.359	0.271	0.927	1.024
B-splines	4	4	0.567	0.398	0.360	0.320	0.221	0.275	0.448	0.242	0.542	0.690
Partitioning	4	4	0.559	0.327	0.360	0.256	0.251	0.568	0.469	0.478	0.473	0.519
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.146	0.112	0.106	0.088	0.120	0.114	0.199	0.147	0.498	0.194
B-splines	4	1	0.106	0.157	0.081	0.117	0.181	0.122	0.156	0.223	0.252	0.528
Partitioning	4	1	0.119	0.137	0.093	0.105	0.216	0.119	0.164	0.197	0.182	0.313
Feasible Estimation												
Local Polynomial	0.22	0.29	0.100	0.087	0.082	0.062	0.145	0.060	0.123	0.135	0.123	0.135
B-splines	4	1	0.106	0.158	0.081	0.117	0.181	0.121	0.156	0.226	0.252	0.547
Partitioning	4	1	0.119	0.147	0.093	0.111	0.216	0.187	0.164	0.224	0.182	0.335
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.058	0.086	0.045	0.064	0.033	0.052	0.071	0.122	0.101	0.179
B-splines	1	1	0.063	0.126	0.047	0.088	0.033	0.060	0.067	0.164	0.137	0.494
Partitioning	1	1	0.055	0.099	0.043	0.072	0.033	0.052	0.067	0.130	0.088	0.278
Feasible Estimation												
Local Polynomial	0.4	0.3	0.353	0.358	0.296	0.299	0.038	0.058	0.115	0.140	0.652	0.658
B-splines	1	1	0.070	0.126	0.051	0.088	0.051	0.060	0.088	0.164	0.172	0.496
Partitioning	1	1	0.068	0.099	0.050	0.073	0.069	0.055	0.092	0.130	0.111	0.279
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.177	0.205	0.097	0.131	0.950	0.910	0.363	0.355	0.294	0.371
B-splines	9	9	0.198	0.241	0.123	0.161	0.950	0.920	0.369	0.397	0.388	0.946
Partitioning	9	9	0.222	0.330	0.154	0.245	0.931	0.855	0.372	0.412	0.290	0.777
Feasible Estimation												
Local Polynomial	0.33	0.27	0.166	0.178	0.078	0.094	0.969	0.969	0.362	0.370	0.116	0.137
B-splines	3	2	0.174	0.204	0.099	0.127	0.926	0.930	0.363	0.386	0.232	0.551
Partitioning	3	2	0.179	0.199	0.104	0.125	0.917	0.890	0.340	0.393	0.170	0.368
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.32	0.9	0.096	0.089	0.067	0.067	0.048	0.058	0.134	0.123	0.309	0.179
B-splines	4	1	0.095	0.126	0.071	0.089	0.106	0.062	0.155	0.165	0.252	0.494
Partitioning	4	1	0.111	0.101	0.086	0.074	0.154	0.058	0.163	0.130	0.182	0.288
Feasible Estimation												
Local Polynomial	0.34	0.28	0.067	0.092	0.049	0.066	0.042	0.058	0.113	0.135	0.114	0.136
B-splines	3	2	0.091	0.136	0.067	0.095	0.092	0.074	0.135	0.184	0.232	0.545
Partitioning	3	2	0.101	0.135	0.076	0.093	0.135	0.214	0.145	0.236	0.166	0.356

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.34: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.21	0.27	0.267	0.345	0.205	0.266	0.240	0.305	0.271	0.327	0.339	0.370
B-splines	4	4	0.228	0.325	0.177	0.250	0.504	0.342	0.300	0.319	0.271	0.333
Partitioning	4	4	0.256	0.405	0.198	0.308	0.472	1.301	0.376	0.766	0.251	0.339
Feasible Estimation												
Local Polynomial	0.29	0.26	0.225	0.252	0.171	0.198	0.119	0.200	0.159	0.193	0.153	0.184
B-splines	3	2	0.233	0.299	0.180	0.230	0.463	0.303	0.285	0.265	0.270	0.326
Partitioning	3	2	0.255	0.310	0.196	0.231	0.446	0.725	0.349	0.461	0.247	0.308
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.15	0.21	0.343	0.412	0.266	0.319	0.314	0.361	0.349	0.440	0.361	0.516
B-splines	9	4	0.425	0.325	0.348	0.249	0.489	0.293	0.428	0.313	0.336	0.334
Partitioning	9	4	0.392	0.411	0.309	0.313	0.332	1.402	0.322	0.772	0.309	0.340
Feasible Estimation												
Local Polynomial	0.31	0.24	0.498	0.512	0.399	0.407	0.709	0.757	0.616	0.626	0.304	0.323
B-splines	4	4	0.382	0.326	0.305	0.250	0.300	0.295	0.284	0.313	0.327	0.334
Partitioning	4	4	0.420	0.411	0.338	0.313	0.941	1.399	0.384	0.767	0.376	0.341
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.19	0.43	0.279	0.271	0.213	0.208	0.219	0.176	0.286	0.223	0.352	0.300
B-splines	9	4	0.288	0.318	0.225	0.244	0.187	0.294	0.227	0.314	0.321	0.333
Partitioning	9	4	0.351	0.404	0.273	0.307	0.238	1.308	0.278	0.766	0.304	0.339
Feasible Estimation												
Local Polynomial	0.32	0.27	0.279	0.305	0.186	0.214	0.142	0.178	0.263	0.288	0.218	0.246
B-splines	4	3	0.250	0.304	0.192	0.232	0.320	0.257	0.286	0.280	0.267	0.328
Partitioning	4	3	0.274	0.344	0.212	0.255	0.368	0.960	0.359	0.566	0.240	0.332
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.686	0.422	0.501	0.332	0.175	0.259	0.351	0.295	0.325	0.382
B-splines	9	9	0.759	0.475	0.603	0.373	0.255	0.350	0.254	0.307	0.337	0.351
Partitioning	9	9	0.712	0.626	0.568	0.478	0.251	0.466	0.311	0.472	0.310	0.406
Feasible Estimation												
Local Polynomial	0.2	0.27	0.648	0.554	0.532	0.452	0.417	0.423	0.669	0.393	0.613	0.921
B-splines	4	2	0.808	0.562	0.625	0.443	0.704	0.428	0.293	0.312	0.297	0.343
Partitioning	4	2	0.801	0.560	0.620	0.438	0.782	1.101	0.340	0.627	0.272	0.328
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.24	0.9	0.243	0.199	0.187	0.158	0.243	0.207	0.246	0.168	0.325	0.215
B-splines	4	1	0.197	0.278	0.151	0.215	0.319	0.234	0.279	0.234	0.271	0.326
Partitioning	4	1	0.225	0.230	0.175	0.181	0.408	0.214	0.303	0.200	0.244	0.285
Feasible Estimation												
Local Polynomial	0.26	0.27	0.152	0.176	0.120	0.136	0.223	0.151	0.148	0.181	0.148	0.181
B-splines	4	2	0.197	0.291	0.151	0.223	0.319	0.255	0.279	0.258	0.271	0.326
Partitioning	4	2	0.225	0.290	0.175	0.216	0.408	0.644	0.303	0.413	0.244	0.302
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.119	0.181	0.092	0.141	0.066	0.132	0.113	0.168	0.165	0.215
B-splines	1	1	0.127	0.255	0.097	0.194	0.066	0.161	0.110	0.233	0.188	0.324
Partitioning	1	1	0.110	0.202	0.086	0.156	0.066	0.133	0.110	0.199	0.144	0.284
Feasible Estimation												
Local Polynomial	0.35	0.28	0.470	0.486	0.410	0.418	0.093	0.148	0.149	0.181	0.660	0.671
B-splines	2	1	0.163	0.260	0.122	0.197	0.196	0.171	0.221	0.237	0.234	0.324
Partitioning	2	1	0.175	0.223	0.127	0.167	0.273	0.323	0.230	0.259	0.198	0.288
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.227	0.341	0.160	0.259	0.967	0.944	0.394	0.431	0.301	0.376
B-splines	9	9	0.283	0.399	0.210	0.303	0.974	0.964	0.404	0.435	0.320	0.342
Partitioning	9	9	0.352	0.611	0.268	0.463	0.956	0.977	0.423	0.558	0.296	0.398
Feasible Estimation												
Local Polynomial	0.32	0.27	0.172	0.215	0.107	0.150	0.989	0.997	0.381	0.393	0.147	0.181
B-splines	3	2	0.213	0.299	0.147	0.220	0.981	0.969	0.415	0.426	0.251	0.326
Partitioning	3	2	0.230	0.296	0.159	0.208	0.989	1.100	0.419	0.512	0.221	0.301
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.37	0.9	0.183	0.183	0.138	0.144	0.109	0.139	0.189	0.169	0.288	0.215
B-splines	1	1	0.138	0.255	0.106	0.194	0.069	0.162	0.112	0.233	0.188	0.324
Partitioning	1	1	0.123	0.203	0.096	0.157	0.069	0.141	0.112	0.200	0.144	0.285
Feasible Estimation												
Local Polynomial	0.32	0.27	0.130	0.183	0.100	0.142	0.095	0.148	0.147	0.181	0.147	0.182
B-splines	3	2	0.183	0.278	0.139	0.210	0.247	0.212	0.256	0.264	0.257	0.326
Partitioning	3	2	0.206	0.285	0.156	0.205	0.342	0.707	0.270	0.435	0.229	0.305

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.35: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.2	0.26	0.275	0.342	0.207	0.258	0.217	0.265	0.307	0.339	0.460	0.450
B-splines	4	4	0.236	0.324	0.183	0.244	0.453	0.303	0.335	0.364	0.336	0.480
Partitioning	4	4	0.260	0.402	0.202	0.305	0.451	1.135	0.395	0.783	0.281	0.462
Feasible Estimation												
Local Polynomial	0.29	0.26	0.239	0.261	0.187	0.208	0.094	0.170	0.178	0.204	0.177	0.200
B-splines	4	2	0.240	0.304	0.187	0.232	0.430	0.274	0.321	0.312	0.330	0.463
Partitioning	4	2	0.261	0.318	0.202	0.238	0.431	0.643	0.373	0.516	0.279	0.394
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.15	0.21	0.337	0.402	0.256	0.303	0.272	0.299	0.376	0.431	0.510	0.620
B-splines	9	4	0.419	0.322	0.340	0.241	0.447	0.255	0.399	0.363	0.459	0.482
Partitioning	9	4	0.389	0.408	0.305	0.309	0.295	1.202	0.331	0.788	0.398	0.461
Feasible Estimation												
Local Polynomial	0.3	0.24	0.516	0.529	0.419	0.427	0.677	0.715	0.592	0.600	0.343	0.360
B-splines	4	4	0.365	0.323	0.285	0.241	0.252	0.256	0.321	0.363	0.446	0.482
Partitioning	4	4	0.414	0.408	0.330	0.309	0.825	1.198	0.428	0.787	0.498	0.463
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.2	0.44	0.268	0.255	0.198	0.191	0.172	0.153	0.305	0.238	0.464	0.350
B-splines	9	4	0.281	0.316	0.216	0.236	0.162	0.255	0.252	0.363	0.419	0.480
Partitioning	9	4	0.345	0.402	0.268	0.304	0.194	1.143	0.308	0.786	0.365	0.464
Feasible Estimation												
Local Polynomial	0.32	0.27	0.233	0.264	0.155	0.183	0.127	0.152	0.257	0.272	0.218	0.236
B-splines	4	3	0.235	0.300	0.177	0.224	0.263	0.219	0.309	0.321	0.330	0.470
Partitioning	4	3	0.258	0.336	0.198	0.246	0.325	0.780	0.355	0.602	0.267	0.427
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.635	0.420	0.448	0.327	0.166	0.225	0.266	0.302	0.390	0.436
B-splines	9	9	0.711	0.476	0.538	0.370	0.173	0.313	0.255	0.345	0.473	0.520
Partitioning	9	9	0.681	0.625	0.526	0.478	0.212	0.377	0.324	0.499	0.392	0.540
Feasible Estimation												
Local Polynomial	0.21	0.27	0.618	0.531	0.495	0.421	0.365	0.367	0.567	0.364	0.732	0.967
B-splines	4	2	0.745	0.536	0.540	0.418	0.539	0.388	0.361	0.364	0.388	0.493
Partitioning	4	2	0.745	0.529	0.547	0.407	0.608	1.034	0.397	0.693	0.350	0.451
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.24	0.9	0.244	0.195	0.184	0.153	0.216	0.184	0.271	0.189	0.419	0.245
B-splines	4	1	0.196	0.275	0.149	0.208	0.288	0.206	0.302	0.267	0.335	0.456
Partitioning	4	1	0.224	0.228	0.175	0.176	0.364	0.189	0.307	0.225	0.272	0.348
Feasible Estimation												
Local Polynomial	0.25	0.27	0.152	0.173	0.121	0.131	0.205	0.129	0.172	0.195	0.172	0.195
B-splines	4	2	0.196	0.288	0.149	0.216	0.288	0.216	0.302	0.302	0.335	0.462
Partitioning	4	2	0.224	0.288	0.175	0.211	0.364	0.529	0.307	0.453	0.272	0.377
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.118	0.178	0.091	0.137	0.066	0.121	0.120	0.187	0.179	0.243
B-splines	1	1	0.127	0.254	0.097	0.188	0.065	0.143	0.115	0.258	0.218	0.453
Partitioning	1	1	0.110	0.201	0.086	0.153	0.065	0.121	0.115	0.214	0.154	0.343
Feasible Estimation												
Local Polynomial	0.34	0.28	0.431	0.449	0.368	0.379	0.084	0.127	0.170	0.197	0.663	0.673
B-splines	2	1	0.164	0.261	0.122	0.193	0.181	0.155	0.233	0.271	0.292	0.451
Partitioning	2	1	0.176	0.230	0.129	0.168	0.247	0.350	0.241	0.303	0.225	0.356
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.232	0.337	0.159	0.249	0.963	0.931	0.404	0.437	0.366	0.431
B-splines	9	9	0.287	0.400	0.210	0.296	0.969	0.947	0.418	0.467	0.419	0.507
Partitioning	9	9	0.354	0.611	0.271	0.465	0.951	0.912	0.454	0.577	0.357	0.533
Feasible Estimation												
Local Polynomial	0.32	0.27	0.180	0.221	0.109	0.149	0.981	0.985	0.389	0.399	0.169	0.195
B-splines	3	2	0.222	0.305	0.153	0.220	0.955	0.959	0.442	0.461	0.313	0.462
Partitioning	3	2	0.239	0.306	0.168	0.214	0.973	1.037	0.427	0.571	0.253	0.376
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.38	0.9	0.177	0.180	0.130	0.139	0.095	0.126	0.194	0.187	0.336	0.243
B-splines	1	1	0.136	0.254	0.104	0.188	0.070	0.144	0.119	0.258	0.219	0.453
Partitioning	1	1	0.121	0.202	0.094	0.154	0.070	0.126	0.118	0.215	0.155	0.346
Feasible Estimation												
Local Polynomial	0.32	0.27	0.125	0.179	0.095	0.135	0.086	0.128	0.169	0.195	0.169	0.196
B-splines	3	2	0.182	0.277	0.137	0.204	0.220	0.178	0.278	0.305	0.315	0.465
Partitioning	3	2	0.205	0.285	0.156	0.203	0.302	0.584	0.283	0.488	0.253	0.383

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.36: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators $d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.19	0.26	0.285	0.334	0.204	0.238	0.186	0.217	0.397	0.397	0.996	0.883
B-splines	9	4	0.298	0.325	0.232	0.230	0.220	0.240	0.304	0.463	0.777	1.265
Partitioning	9	4	0.340	0.403	0.262	0.300	0.196	0.877	0.348	0.925	0.587	0.985
Feasible Estimation												
Local Polynomial	0.28	0.26	0.252	0.268	0.203	0.214	0.083	0.141	0.239	0.277	0.248	0.279
B-splines	4	3	0.245	0.316	0.190	0.230	0.369	0.247	0.405	0.422	0.506	1.160
Partitioning	4	3	0.265	0.356	0.207	0.262	0.433	0.673	0.466	0.718	0.369	0.848
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.16	0.21	0.332	0.383	0.238	0.272	0.227	0.228	0.467	0.481	1.373	1.300
B-splines	9	4	0.395	0.320	0.312	0.225	0.391	0.205	0.371	0.460	0.880	1.265
Partitioning	9	4	0.380	0.407	0.295	0.303	0.264	0.894	0.365	0.936	0.705	0.988
Feasible Estimation												
Local Polynomial	0.25	0.23	0.524	0.537	0.428	0.438	0.610	0.624	0.557	0.570	0.435	0.460
B-splines	4	4	0.324	0.320	0.242	0.225	0.235	0.207	0.433	0.460	0.744	1.260
Partitioning	4	4	0.386	0.407	0.299	0.303	0.668	0.894	0.492	0.934	0.734	0.990
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.22	0.48	0.245	0.226	0.169	0.162	0.124	0.123	0.348	0.290	0.842	0.584
B-splines	4	4	0.216	0.316	0.158	0.221	0.218	0.206	0.346	0.460	0.508	1.265
Partitioning	4	4	0.242	0.402	0.185	0.299	0.302	0.876	0.369	0.930	0.373	0.991
Feasible Estimation												
Local Polynomial	0.31	0.27	0.178	0.216	0.124	0.151	0.111	0.131	0.282	0.306	0.254	0.286
B-splines	3	3	0.211	0.302	0.153	0.211	0.205	0.189	0.342	0.421	0.486	1.192
Partitioning	3	3	0.233	0.355	0.175	0.254	0.288	0.716	0.358	0.768	0.357	0.889
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.521	0.399	0.345	0.303	0.149	0.183	0.319	0.338	0.690	0.748
B-splines	9	9	0.597	0.476	0.410	0.355	0.103	0.259	0.429	0.456	0.901	2.033
Partitioning	9	9	0.598	0.625	0.433	0.468	0.167	0.275	0.482	0.577	0.704	1.562
Feasible Estimation												
Local Polynomial	0.23	0.27	0.545	0.477	0.412	0.363	0.292	0.284	0.351	0.355	1.036	1.057
B-splines	4	3	0.596	0.492	0.382	0.377	0.322	0.312	0.535	0.478	0.671	1.286
Partitioning	4	3	0.604	0.485	0.403	0.369	0.394	0.904	0.578	0.889	0.557	0.971
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.23	0.9	0.243	0.186	0.175	0.141	0.183	0.145	0.329	0.256	0.789	0.363
B-splines	4	1	0.195	0.268	0.146	0.192	0.258	0.160	0.310	0.361	0.504	1.006
Partitioning	4	1	0.223	0.219	0.173	0.163	0.341	0.149	0.326	0.298	0.363	0.573
Feasible Estimation												
Local Polynomial	0.25	0.27	0.154	0.174	0.121	0.125	0.183	0.118	0.240	0.272	0.240	0.272
B-splines	4	2	0.195	0.295	0.146	0.208	0.258	0.181	0.310	0.412	0.504	1.139
Partitioning	4	2	0.223	0.325	0.173	0.231	0.341	0.589	0.326	0.652	0.363	0.815
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.116	0.172	0.089	0.129	0.065	0.104	0.142	0.244	0.202	0.357
B-splines	1	1	0.126	0.251	0.094	0.177	0.065	0.119	0.134	0.327	0.274	0.988
Partitioning	1	1	0.110	0.198	0.086	0.145	0.065	0.103	0.134	0.259	0.175	0.557
Feasible Estimation												
Local Polynomial	0.34	0.28	0.367	0.388	0.305	0.319	0.077	0.117	0.230	0.273	0.687	0.703
B-splines	3	2	0.172	0.270	0.125	0.188	0.173	0.146	0.264	0.369	0.455	1.089
Partitioning	3	2	0.188	0.267	0.139	0.184	0.255	0.428	0.275	0.470	0.320	0.704
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.238	0.323	0.156	0.226	0.952	0.916	0.424	0.455	0.588	0.743
B-splines	9	9	0.295	0.408	0.210	0.287	0.954	0.929	0.440	0.551	0.775	1.889
Partitioning	9	9	0.360	0.615	0.273	0.461	0.938	0.886	0.474	0.640	0.581	1.554
Feasible Estimation												
Local Polynomial	0.31	0.26	0.196	0.234	0.116	0.151	0.970	0.971	0.411	0.435	0.234	0.273
B-splines	3	3	0.235	0.328	0.160	0.227	0.927	0.928	0.440	0.522	0.490	1.156
Partitioning	3	3	0.254	0.359	0.180	0.252	0.928	1.002	0.425	0.734	0.353	0.840
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.4	0.9	0.165	0.173	0.118	0.130	0.078	0.107	0.221	0.245	0.475	0.357
B-splines	1	1	0.132	0.252	0.099	0.177	0.072	0.120	0.138	0.328	0.276	0.988
Partitioning	1	1	0.117	0.199	0.091	0.145	0.072	0.107	0.139	0.259	0.177	0.562
Feasible Estimation												
Local Polynomial	0.31	0.27	0.123	0.177	0.090	0.127	0.080	0.118	0.231	0.273	0.232	0.273
B-splines	3	3	0.183	0.291	0.134	0.202	0.195	0.170	0.289	0.403	0.486	1.153
Partitioning	3	3	0.207	0.333	0.157	0.233	0.283	0.629	0.305	0.692	0.348	0.840

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

C.2.2 Quantile Cell Boundaries

Table C.37: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.19	0.25	0.207	0.255	0.158	0.197	0.199	0.234	0.213	0.250	0.249	0.288
B-splines	9	4	0.216	0.233	0.169	0.180	0.198	0.271	0.152	0.231	0.232	0.242
Partitioning	9	4	0.245	0.287	0.190	0.218	0.207	0.782	0.187	0.461	0.214	0.242
Feasible Estimation												
Local Polynomial	0.3	0.27	0.207	0.217	0.155	0.170	0.088	0.161	0.122	0.145	0.112	0.134
B-splines	3	1	0.198	0.218	0.150	0.170	0.354	0.261	0.172	0.177	0.180	0.231
Partitioning	3	1	0.208	0.207	0.158	0.159	0.311	0.279	0.248	0.203	0.167	0.204
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.262	0.309	0.204	0.239	0.250	0.284	0.261	0.347	0.258	0.411
B-splines	9	4	0.402	0.234	0.329	0.180	0.522	0.206	0.423	0.226	0.246	0.243
Partitioning	9	4	0.317	0.295	0.250	0.225	0.333	0.880	0.275	0.467	0.218	0.245
Feasible Estimation												
Local Polynomial	0.33	0.24	0.490	0.494	0.392	0.391	0.706	0.757	0.607	0.615	0.287	0.297
B-splines	4	4	0.356	0.234	0.285	0.180	0.252	0.206	0.212	0.227	0.262	0.243
Partitioning	4	4	0.386	0.295	0.312	0.225	0.853	0.880	0.313	0.466	0.323	0.244
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.17	0.4	0.216	0.204	0.165	0.156	0.178	0.133	0.227	0.176	0.257	0.221
B-splines	9	4	0.217	0.224	0.169	0.171	0.142	0.204	0.171	0.226	0.233	0.242
Partitioning	9	4	0.255	0.286	0.198	0.217	0.145	0.774	0.194	0.462	0.223	0.243
Feasible Estimation												
Local Polynomial	0.34	0.28	0.264	0.276	0.170	0.185	0.118	0.137	0.247	0.259	0.198	0.212
B-splines	4	2	0.209	0.220	0.160	0.170	0.223	0.182	0.200	0.194	0.183	0.238
Partitioning	4	2	0.226	0.234	0.174	0.176	0.256	0.433	0.294	0.306	0.159	0.234
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.668	0.354	0.482	0.275	0.149	0.207	0.323	0.206	0.266	0.271
B-splines	9	9	0.693	0.379	0.555	0.296	0.184	0.272	0.238	0.230	0.290	0.266
Partitioning	9	9	0.590	0.444	0.455	0.341	0.188	0.300	0.277	0.323	0.260	0.337
Feasible Estimation												
Local Polynomial	0.21	0.28	0.638	0.540	0.524	0.443	0.405	0.412	0.668	0.379	0.601	0.907
B-splines	4	1	0.784	0.567	0.609	0.447	0.584	0.395	0.219	0.245	0.234	0.286
Partitioning	4	1	0.769	0.556	0.590	0.434	0.661	0.589	0.259	0.333	0.213	0.247
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.22	0.9	0.186	0.151	0.142	0.120	0.195	0.181	0.192	0.121	0.238	0.145
B-splines	4	1	0.143	0.209	0.111	0.164	0.201	0.200	0.185	0.170	0.182	0.234
Partitioning	4	1	0.161	0.178	0.126	0.142	0.272	0.188	0.214	0.146	0.159	0.188
Feasible Estimation												
Local Polynomial	0.26	0.29	0.122	0.122	0.096	0.093	0.206	0.104	0.105	0.128	0.105	0.128
B-splines	4	1	0.143	0.210	0.111	0.164	0.201	0.200	0.185	0.170	0.182	0.235
Partitioning	4	1	0.161	0.182	0.126	0.144	0.272	0.205	0.214	0.152	0.159	0.189
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.099	0.044	0.091	0.080	0.119	0.110	0.144
B-splines	1	1	0.089	0.178	0.068	0.136	0.044	0.114	0.077	0.164	0.129	0.230
Partitioning	1	1	0.076	0.140	0.060	0.109	0.044	0.092	0.076	0.140	0.097	0.185
Feasible Estimation												
Local Polynomial	0.4	0.31	0.461	0.468	0.405	0.407	0.062	0.110	0.112	0.139	0.647	0.658
B-splines	1	1	0.092	0.178	0.070	0.136	0.060	0.114	0.085	0.164	0.134	0.230
Partitioning	1	1	0.082	0.140	0.062	0.109	0.064	0.092	0.088	0.140	0.101	0.185
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.181	0.254	0.119	0.188	0.960	0.931	0.378	0.384	0.208	0.265
B-splines	9	9	0.216	0.293	0.153	0.218	0.965	0.949	0.382	0.403	0.232	0.254
Partitioning	9	9	0.261	0.437	0.195	0.334	0.952	0.932	0.393	0.447	0.212	0.336
Feasible Estimation												
Local Polynomial	0.34	0.28	0.148	0.173	0.079	0.110	0.981	0.983	0.369	0.374	0.104	0.128
B-splines	2	1	0.164	0.217	0.099	0.153	0.964	0.960	0.378	0.387	0.153	0.230
Partitioning	2	1	0.166	0.190	0.099	0.128	0.950	0.958	0.370	0.377	0.126	0.187
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.33	0.9	0.137	0.130	0.104	0.102	0.086	0.103	0.145	0.119	0.210	0.145
B-splines	1	1	0.104	0.179	0.079	0.137	0.050	0.116	0.078	0.164	0.130	0.230
Partitioning	1	1	0.094	0.142	0.072	0.110	0.050	0.105	0.078	0.143	0.097	0.186
Feasible Estimation												
Local Polynomial	0.34	0.28	0.097	0.132	0.073	0.101	0.062	0.102	0.104	0.128	0.104	0.128
B-splines	2	1	0.123	0.183	0.093	0.140	0.133	0.123	0.149	0.170	0.162	0.232
Partitioning	2	1	0.133	0.160	0.100	0.119	0.192	0.222	0.167	0.172	0.136	0.191

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.38: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.18	0.25	0.214	0.256	0.162	0.194	0.184	0.211	0.237	0.264	0.376	0.386
B-splines	9	4	0.225	0.236	0.177	0.179	0.222	0.243	0.177	0.256	0.329	0.398
Partitioning	9	4	0.247	0.286	0.192	0.218	0.194	0.640	0.221	0.525	0.277	0.367
Feasible Estimation												
Local Polynomial	0.29	0.27	0.224	0.231	0.173	0.184	0.072	0.145	0.134	0.159	0.135	0.150
B-splines	3	1	0.205	0.229	0.158	0.178	0.355	0.253	0.227	0.209	0.252	0.359
Partitioning	3	1	0.215	0.221	0.166	0.170	0.336	0.306	0.312	0.286	0.210	0.288
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.262	0.303	0.200	0.229	0.224	0.238	0.288	0.349	0.390	0.555
B-splines	9	4	0.375	0.234	0.306	0.175	0.436	0.184	0.360	0.253	0.404	0.400
Partitioning	9	4	0.312	0.294	0.246	0.223	0.260	0.717	0.256	0.535	0.342	0.365
Feasible Estimation												
Local Polynomial	0.3	0.24	0.508	0.513	0.413	0.414	0.671	0.701	0.589	0.586	0.316	0.335
B-splines	4	4	0.339	0.234	0.265	0.175	0.196	0.184	0.221	0.253	0.407	0.400
Partitioning	4	4	0.381	0.294	0.306	0.224	0.755	0.717	0.342	0.535	0.474	0.365
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.18	0.41	0.209	0.194	0.155	0.145	0.146	0.119	0.238	0.185	0.385	0.294
B-splines	9	4	0.217	0.225	0.167	0.167	0.133	0.183	0.192	0.252	0.335	0.399
Partitioning	9	4	0.255	0.285	0.198	0.217	0.148	0.631	0.230	0.530	0.287	0.371
Feasible Estimation												
Local Polynomial	0.34	0.28	0.217	0.233	0.141	0.155	0.114	0.126	0.233	0.246	0.188	0.204
B-splines	3	2	0.195	0.218	0.144	0.164	0.177	0.165	0.229	0.220	0.244	0.381
Partitioning	3	2	0.208	0.233	0.157	0.173	0.228	0.407	0.278	0.375	0.194	0.333
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.615	0.355	0.429	0.276	0.148	0.186	0.223	0.212	0.336	0.340
B-splines	9	9	0.679	0.392	0.509	0.308	0.157	0.276	0.214	0.254	0.435	0.483
Partitioning	9	9	0.630	0.458	0.477	0.352	0.171	0.294	0.263	0.386	0.345	0.476
Feasible Estimation												
Local Polynomial	0.22	0.28	0.604	0.516	0.485	0.411	0.362	0.365	0.547	0.344	0.734	0.953
B-splines	4	1	0.724	0.499	0.523	0.388	0.451	0.344	0.284	0.266	0.369	0.435
Partitioning	4	1	0.717	0.477	0.521	0.361	0.512	0.587	0.345	0.467	0.336	0.370
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.21	0.9	0.189	0.149	0.143	0.118	0.178	0.163	0.208	0.140	0.345	0.183
B-splines	4	1	0.145	0.208	0.112	0.160	0.190	0.176	0.204	0.197	0.255	0.366
Partitioning	4	1	0.163	0.178	0.127	0.140	0.263	0.170	0.214	0.167	0.205	0.269
Feasible Estimation												
Local Polynomial	0.26	0.29	0.127	0.125	0.102	0.094	0.198	0.095	0.126	0.146	0.126	0.146
B-splines	4	1	0.145	0.210	0.112	0.160	0.190	0.176	0.204	0.203	0.255	0.367
Partitioning	4	1	0.163	0.185	0.127	0.144	0.263	0.203	0.214	0.200	0.205	0.272
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.097	0.046	0.087	0.088	0.137	0.128	0.181
B-splines	1	1	0.090	0.180	0.069	0.134	0.045	0.102	0.084	0.183	0.159	0.359
Partitioning	1	1	0.078	0.142	0.061	0.109	0.045	0.088	0.085	0.153	0.108	0.263
Feasible Estimation												
Local Polynomial	0.4	0.31	0.422	0.430	0.363	0.367	0.058	0.094	0.128	0.153	0.662	0.662
B-splines	1	1	0.094	0.180	0.071	0.134	0.057	0.102	0.098	0.183	0.168	0.359
Partitioning	1	1	0.086	0.142	0.065	0.109	0.083	0.088	0.102	0.153	0.115	0.263
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.189	0.255	0.120	0.183	0.959	0.926	0.376	0.391	0.284	0.333
B-splines	9	9	0.224	0.298	0.157	0.218	0.960	0.938	0.383	0.406	0.331	0.446
Partitioning	9	9	0.267	0.440	0.199	0.335	0.944	0.891	0.402	0.480	0.275	0.466
Feasible Estimation												
Local Polynomial	0.34	0.28	0.159	0.184	0.084	0.113	0.982	0.986	0.367	0.380	0.122	0.146
B-splines	2	1	0.176	0.227	0.107	0.156	0.958	0.954	0.384	0.395	0.210	0.361
Partitioning	2	1	0.179	0.207	0.110	0.138	0.944	0.953	0.359	0.402	0.158	0.274
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.34	0.9	0.136	0.129	0.100	0.100	0.077	0.093	0.153	0.138	0.283	0.181
B-splines	1	1	0.102	0.180	0.078	0.134	0.052	0.104	0.087	0.183	0.159	0.358
Partitioning	1	1	0.092	0.144	0.071	0.110	0.052	0.095	0.088	0.154	0.110	0.269
Feasible Estimation												
Local Polynomial	0.34	0.28	0.094	0.132	0.071	0.099	0.060	0.091	0.121	0.146	0.122	0.146
B-splines	3	1	0.125	0.186	0.094	0.138	0.122	0.112	0.169	0.194	0.221	0.361
Partitioning	3	1	0.135	0.165	0.101	0.122	0.190	0.204	0.175	0.234	0.169	0.281

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.39: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators $d = 2$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.17	0.24	0.232	0.249	0.159	0.179	0.151	0.161	0.336	0.335	1.295	1.400
B-splines	9	4	0.234	0.236	0.184	0.170	0.231	0.187	0.290	0.375	0.517	1.160
Partitioning	9	4	0.248	0.286	0.193	0.214	0.166	0.556	0.301	0.672	0.385	0.792
Feasible Estimation												
Local Polynomial	0.27	0.26	0.236	0.237	0.191	0.191	0.058	0.113	0.172	0.204	0.176	0.198
B-splines	4	2	0.207	0.239	0.162	0.182	0.316	0.223	0.340	0.340	0.369	0.937
Partitioning	4	2	0.217	0.247	0.169	0.189	0.354	0.383	0.396	0.448	0.263	0.601
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.14	0.2	0.259	0.291	0.186	0.206	0.180	0.169	0.379	0.425	4.109	7.040
B-splines	9	4	0.311	0.229	0.243	0.163	0.304	0.144	0.257	0.366	0.753	1.157
Partitioning	9	4	0.301	0.290	0.234	0.217	0.189	0.586	0.292	0.680	0.662	0.799
Feasible Estimation												
Local Polynomial	0.25	0.23	0.516	0.521	0.422	0.425	0.605	0.611	0.516	0.526	0.420	0.437
B-splines	4	4	0.292	0.229	0.218	0.163	0.172	0.144	0.341	0.366	0.708	1.157
Partitioning	4	4	0.347	0.290	0.269	0.217	0.580	0.586	0.424	0.680	0.697	0.801
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.19	0.45	0.189	0.170	0.131	0.122	0.097	0.093	0.295	0.230	0.817	0.440
B-splines	9	4	0.202	0.224	0.150	0.158	0.106	0.144	0.267	0.366	0.520	1.156
Partitioning	9	4	0.245	0.284	0.189	0.212	0.144	0.539	0.318	0.677	0.366	0.801
Feasible Estimation												
Local Polynomial	0.33	0.27	0.156	0.176	0.107	0.122	0.095	0.100	0.246	0.254	0.207	0.224
B-splines	3	3	0.165	0.215	0.118	0.152	0.138	0.127	0.275	0.338	0.344	1.003
Partitioning	3	3	0.176	0.242	0.129	0.174	0.196	0.415	0.286	0.526	0.239	0.686
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.501	0.342	0.326	0.262	0.137	0.150	0.286	0.263	0.588	0.600
B-splines	9	9	0.569	0.399	0.379	0.309	0.098	0.245	0.431	0.475	0.736	2.468
Partitioning	9	9	0.571	0.463	0.401	0.354	0.152	0.294	0.475	0.591	0.550	1.355
Feasible Estimation												
Local Polynomial	0.23	0.28	0.529	0.459	0.400	0.350	0.284	0.279	0.345	0.326	0.991	1.024
B-splines	4	2	0.575	0.430	0.363	0.337	0.252	0.280	0.487	0.396	0.604	1.221
Partitioning	4	2	0.578	0.390	0.376	0.300	0.311	0.580	0.524	0.703	0.505	0.843
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.21	0.9	0.188	0.141	0.136	0.109	0.147	0.122	0.276	0.196	0.688	0.262
B-splines	4	1	0.144	0.201	0.109	0.146	0.181	0.135	0.223	0.298	0.372	0.809
Partitioning	4	1	0.162	0.170	0.126	0.127	0.249	0.126	0.226	0.246	0.250	0.426
Feasible Estimation												
Local Polynomial	0.25	0.28	0.126	0.122	0.103	0.088	0.172	0.081	0.169	0.193	0.169	0.193
B-splines	4	1	0.144	0.206	0.109	0.149	0.181	0.133	0.223	0.312	0.372	0.876
Partitioning	4	1	0.162	0.195	0.126	0.142	0.249	0.239	0.226	0.343	0.250	0.525
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.081	0.122	0.062	0.091	0.043	0.074	0.097	0.172	0.136	0.244
B-splines	1	1	0.090	0.178	0.067	0.126	0.043	0.085	0.094	0.250	0.194	0.788
Partitioning	1	1	0.077	0.141	0.059	0.104	0.043	0.073	0.094	0.187	0.122	0.394
Feasible Estimation												
Local Polynomial	0.39	0.3	0.356	0.366	0.297	0.304	0.052	0.083	0.159	0.207	0.658	0.667
B-splines	1	1	0.099	0.179	0.072	0.126	0.064	0.086	0.124	0.250	0.246	0.789
Partitioning	1	1	0.093	0.143	0.068	0.104	0.095	0.079	0.124	0.193	0.157	0.398
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.199	0.250	0.120	0.171	0.951	0.914	0.393	0.414	0.413	0.587
B-splines	9	9	0.233	0.306	0.160	0.216	0.943	0.918	0.401	0.504	0.512	2.002
Partitioning	9	9	0.273	0.442	0.203	0.335	0.918	0.827	0.443	0.656	0.361	1.323
Feasible Estimation												
Local Polynomial	0.33	0.27	0.176	0.198	0.091	0.117	0.969	0.972	0.387	0.407	0.159	0.194
B-splines	3	2	0.195	0.246	0.121	0.165	0.932	0.934	0.391	0.454	0.337	0.913
Partitioning	3	2	0.201	0.246	0.130	0.166	0.901	0.910	0.357	0.511	0.230	0.567
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.36	0.9	0.125	0.124	0.089	0.093	0.060	0.078	0.175	0.172	0.378	0.244
B-splines	1	1	0.098	0.179	0.073	0.127	0.053	0.087	0.101	0.250	0.195	0.788
Partitioning	1	1	0.086	0.142	0.066	0.105	0.052	0.079	0.101	0.187	0.125	0.400
Feasible Estimation												
Local Polynomial	0.33	0.27	0.089	0.127	0.064	0.090	0.055	0.080	0.157	0.194	0.157	0.193
B-splines	3	2	0.127	0.195	0.093	0.137	0.120	0.104	0.199	0.292	0.332	0.911
Partitioning	3	2	0.139	0.200	0.104	0.139	0.181	0.320	0.201	0.390	0.223	0.560

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.40: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.23	0.29	0.345	0.530	0.264	0.405	0.281	0.383	0.363	0.501	0.462	0.668
B-splines	4	4	0.294	0.451	0.228	0.346	0.515	0.442	0.373	0.454	0.362	0.484
Partitioning	4	4	0.334	0.568	0.260	0.432	0.551	1.551	0.467	0.918	0.324	0.484
Feasible Estimation												
Local Polynomial	0.3	0.26	0.253	0.304	0.196	0.240	0.142	0.244	0.219	0.270	0.214	0.265
B-splines	3	2	0.292	0.409	0.225	0.313	0.467	0.377	0.341	0.390	0.348	0.471
Partitioning	3	2	0.322	0.432	0.248	0.316	0.503	0.931	0.424	0.571	0.309	0.419
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.437	0.549	0.337	0.423	0.382	0.479	0.463	0.566	0.513	0.673
B-splines	9	4	0.507	0.452	0.409	0.346	0.547	0.409	0.495	0.452	0.470	0.484
Partitioning	9	4	0.511	0.573	0.400	0.436	0.408	1.609	0.422	0.922	0.427	0.485
Feasible Estimation												
Local Polynomial	0.31	0.25	0.510	0.538	0.408	0.426	0.721	0.783	0.632	0.658	0.340	0.369
B-splines	4	4	0.426	0.456	0.338	0.351	0.400	0.422	0.388	0.446	0.408	0.483
Partitioning	4	4	0.470	0.565	0.374	0.432	0.947	1.535	0.491	0.865	0.422	0.484
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.21	0.46	0.363	0.360	0.275	0.275	0.271	0.220	0.390	0.304	0.481	0.387
B-splines	4	4	0.310	0.447	0.238	0.342	0.383	0.408	0.369	0.452	0.362	0.483
Partitioning	4	4	0.350	0.568	0.273	0.432	0.498	1.539	0.474	0.918	0.319	0.484
Feasible Estimation												
Local Polynomial	0.31	0.27	0.300	0.348	0.206	0.251	0.161	0.227	0.306	0.343	0.268	0.310
B-splines	3	2	0.306	0.411	0.233	0.313	0.355	0.332	0.370	0.400	0.350	0.474
Partitioning	3	2	0.339	0.451	0.261	0.329	0.460	0.973	0.461	0.636	0.301	0.440
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.707	0.529	0.523	0.415	0.205	0.336	0.410	0.406	0.449	0.533
B-splines	9	9	0.758	0.599	0.605	0.464	0.246	0.399	0.355	0.434	0.496	0.515
Partitioning	9	9	0.713	0.858	0.559	0.657	0.303	0.599	0.427	0.640	0.450	0.671
Feasible Estimation												
Local Polynomial	0.23	0.27	0.685	0.581	0.554	0.469	0.432	0.448	0.671	0.440	0.649	0.939
B-splines	4	2	0.830	0.661	0.639	0.523	0.687	0.487	0.387	0.426	0.389	0.499
Partitioning	4	2	0.834	0.672	0.648	0.533	0.812	1.075	0.450	0.654	0.345	0.440
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.27	0.9	0.314	0.265	0.241	0.209	0.297	0.241	0.323	0.239	0.433	0.288
B-splines	4	1	0.272	0.373	0.209	0.287	0.365	0.281	0.367	0.333	0.362	0.462
Partitioning	4	1	0.312	0.302	0.244	0.235	0.500	0.246	0.426	0.284	0.318	0.370
Feasible Estimation												
Local Polynomial	0.27	0.27	0.195	0.244	0.152	0.188	0.258	0.210	0.209	0.260	0.209	0.260
B-splines	4	2	0.279	0.398	0.214	0.303	0.381	0.319	0.373	0.381	0.369	0.468
Partitioning	4	2	0.318	0.403	0.248	0.294	0.509	0.805	0.429	0.523	0.326	0.406
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.166	0.252	0.129	0.198	0.088	0.183	0.161	0.238	0.221	0.288
B-splines	1	1	0.177	0.357	0.136	0.273	0.087	0.228	0.153	0.329	0.258	0.460
Partitioning	1	1	0.153	0.281	0.119	0.217	0.087	0.183	0.153	0.280	0.193	0.369
Feasible Estimation												
Local Polynomial	0.34	0.28	0.482	0.513	0.416	0.433	0.126	0.213	0.213	0.263	0.671	0.697
B-splines	2	1	0.226	0.365	0.169	0.278	0.242	0.240	0.284	0.340	0.320	0.463
Partitioning	2	1	0.241	0.315	0.176	0.235	0.335	0.426	0.319	0.337	0.263	0.378
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.296	0.468	0.217	0.358	0.970	0.970	0.456	0.519	0.416	0.531
B-splines	9	9	0.377	0.549	0.286	0.418	0.978	0.996	0.467	0.549	0.463	0.509
Partitioning	9	9	0.478	0.854	0.371	0.654	0.982	1.082	0.514	0.711	0.425	0.673
Feasible Estimation												
Local Polynomial	0.31	0.27	0.207	0.274	0.140	0.201	0.985	0.997	0.411	0.435	0.208	0.261
B-splines	3	2	0.274	0.405	0.198	0.304	0.991	0.982	0.474	0.520	0.338	0.466
Partitioning	3	2	0.298	0.406	0.216	0.290	0.992	1.194	0.489	0.620	0.291	0.406
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.42	0.9	0.239	0.254	0.180	0.199	0.129	0.189	0.237	0.238	0.363	0.288
B-splines	1	1	0.185	0.357	0.142	0.273	0.091	0.229	0.154	0.329	0.259	0.460
Partitioning	1	1	0.162	0.282	0.126	0.218	0.091	0.190	0.154	0.282	0.193	0.370
Feasible Estimation												
Local Polynomial	0.31	0.27	0.173	0.249	0.132	0.192	0.129	0.210	0.209	0.261	0.208	0.261
B-splines	3	2	0.251	0.392	0.190	0.297	0.308	0.306	0.330	0.385	0.339	0.468
Partitioning	3	2	0.282	0.408	0.213	0.293	0.424	0.859	0.379	0.551	0.294	0.411

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.41: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.22	0.28	0.358	0.540	0.268	0.400	0.264	0.350	0.404	0.528	0.676	0.883
B-splines	4	4	0.304	0.455	0.235	0.340	0.474	0.399	0.427	0.507	0.506	0.797
Partitioning	4	4	0.340	0.568	0.265	0.431	0.539	1.264	0.507	1.047	0.411	0.731
Feasible Estimation												
Local Polynomial	0.3	0.26	0.269	0.315	0.211	0.248	0.129	0.222	0.253	0.300	0.257	0.296
B-splines	3	2	0.304	0.415	0.236	0.313	0.442	0.344	0.402	0.430	0.493	0.745
Partitioning	3	2	0.332	0.437	0.258	0.322	0.506	0.784	0.477	0.718	0.393	0.610
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.436	0.541	0.329	0.408	0.345	0.408	0.501	0.582	0.762	0.854
B-splines	9	4	0.488	0.454	0.389	0.338	0.464	0.366	0.461	0.505	0.710	0.798
Partitioning	9	4	0.511	0.571	0.399	0.434	0.359	1.310	0.459	1.054	0.593	0.727
Feasible Estimation												
Local Polynomial	0.3	0.25	0.529	0.556	0.429	0.447	0.677	0.717	0.633	0.639	0.376	0.418
B-splines	4	4	0.413	0.457	0.322	0.342	0.345	0.375	0.419	0.491	0.606	0.794
Partitioning	4	4	0.467	0.565	0.370	0.431	0.849	1.230	0.498	0.993	0.597	0.740
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.22	0.47	0.351	0.342	0.259	0.256	0.228	0.201	0.404	0.325	0.684	0.517
B-splines	4	1	0.305	0.377	0.231	0.281	0.340	0.231	0.413	0.374	0.509	0.717
Partitioning	4	1	0.343	0.311	0.266	0.236	0.472	0.186	0.455	0.338	0.408	0.545
Feasible Estimation												
Local Polynomial	0.32	0.27	0.262	0.317	0.181	0.227	0.157	0.207	0.312	0.355	0.281	0.328
B-splines	3	2	0.297	0.415	0.223	0.307	0.307	0.303	0.402	0.440	0.488	0.753
Partitioning	3	2	0.327	0.454	0.249	0.329	0.439	0.867	0.441	0.787	0.388	0.635
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.659	0.523	0.472	0.404	0.197	0.299	0.347	0.420	0.596	0.671
B-splines	9	9	0.747	0.609	0.568	0.467	0.225	0.383	0.358	0.478	0.721	0.911
Partitioning	9	9	0.749	0.867	0.583	0.663	0.303	0.588	0.462	0.763	0.590	0.939
Feasible Estimation												
Local Polynomial	0.23	0.27	0.652	0.560	0.517	0.444	0.392	0.401	0.545	0.428	0.814	0.987
B-splines	4	2	0.765	0.635	0.555	0.491	0.540	0.436	0.449	0.481	0.575	0.795
Partitioning	4	2	0.774	0.649	0.575	0.502	0.654	0.994	0.516	0.854	0.491	0.678
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.27	0.9	0.320	0.264	0.241	0.206	0.273	0.224	0.353	0.275	0.612	0.361
B-splines	4	1	0.276	0.374	0.210	0.281	0.336	0.250	0.406	0.374	0.508	0.719
Partitioning	4	1	0.315	0.304	0.245	0.234	0.479	0.231	0.427	0.314	0.408	0.527
Feasible Estimation												
Local Polynomial	0.27	0.27	0.204	0.249	0.161	0.187	0.252	0.187	0.251	0.294	0.251	0.294
B-splines	4	2	0.281	0.400	0.214	0.298	0.343	0.284	0.413	0.423	0.510	0.740
Partitioning	4	2	0.319	0.406	0.248	0.295	0.484	0.698	0.432	0.663	0.413	0.595
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.166	0.251	0.128	0.194	0.092	0.174	0.177	0.274	0.256	0.361
B-splines	1	1	0.180	0.359	0.137	0.267	0.090	0.204	0.169	0.367	0.318	0.717
Partitioning	1	1	0.156	0.285	0.122	0.217	0.090	0.176	0.169	0.307	0.217	0.526
Feasible Estimation												
Local Polynomial	0.35	0.28	0.446	0.481	0.377	0.399	0.118	0.185	0.249	0.298	0.701	0.711
B-splines	2	1	0.233	0.370	0.172	0.274	0.228	0.221	0.317	0.388	0.426	0.724
Partitioning	2	1	0.248	0.328	0.182	0.241	0.347	0.400	0.330	0.464	0.324	0.550
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.301	0.461	0.216	0.344	0.969	0.956	0.461	0.537	0.568	0.667
B-splines	9	9	0.384	0.553	0.290	0.413	0.974	0.974	0.479	0.572	0.661	0.891
Partitioning	9	9	0.485	0.858	0.376	0.654	0.980	1.028	0.555	0.810	0.550	0.932
Feasible Estimation												
Local Polynomial	0.32	0.27	0.217	0.283	0.145	0.203	0.990	1.003	0.420	0.458	0.247	0.293
B-splines	3	2	0.286	0.415	0.206	0.306	0.973	0.970	0.507	0.551	0.478	0.740
Partitioning	3	2	0.311	0.426	0.228	0.305	0.993	1.137	0.479	0.748	0.376	0.600
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.42	0.9	0.235	0.253	0.174	0.196	0.121	0.177	0.251	0.275	0.480	0.361
B-splines	1	1	0.187	0.359	0.142	0.267	0.094	0.204	0.170	0.367	0.318	0.717
Partitioning	1	1	0.163	0.286	0.127	0.218	0.094	0.179	0.171	0.307	0.217	0.530
Feasible Estimation												
Local Polynomial	0.32	0.27	0.175	0.252	0.132	0.190	0.121	0.186	0.247	0.293	0.247	0.293
B-splines	3	2	0.258	0.397	0.194	0.293	0.284	0.269	0.372	0.431	0.483	0.743
Partitioning	3	2	0.289	0.417	0.219	0.298	0.423	0.749	0.387	0.704	0.378	0.607

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.42: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2,2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.21	0.27	0.366	0.453	0.261	0.324	0.220	0.268	0.554	0.587	1.384	1.542
B-splines	4	4	0.309	0.453	0.236	0.323	0.405	0.311	0.515	0.736	0.738	2.314
Partitioning	4	4	0.342	0.567	0.266	0.424	0.514	1.086	0.562	1.342	0.503	1.574
Feasible Estimation												
Local Polynomial	0.29	0.26	0.279	0.320	0.223	0.247	0.112	0.183	0.332	0.398	0.334	0.392
B-splines	4	3	0.310	0.434	0.237	0.313	0.392	0.304	0.499	0.672	0.724	2.063
Partitioning	4	3	0.340	0.495	0.264	0.362	0.502	0.859	0.542	1.094	0.500	1.359
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.423	0.511	0.303	0.365	0.277	0.290	0.654	0.702	6.256	5.066
B-splines	9	4	0.438	0.450	0.336	0.319	0.339	0.286	0.441	0.731	1.171	2.310
Partitioning	9	4	0.501	0.569	0.389	0.425	0.309	1.103	0.576	1.346	0.908	1.571
Feasible Estimation												
Local Polynomial	0.27	0.24	0.536	0.563	0.436	0.453	0.612	0.628	0.589	0.617	0.506	0.558
B-splines	4	4	0.372	0.450	0.279	0.320	0.287	0.292	0.542	0.721	0.937	2.237
Partitioning	4	4	0.441	0.564	0.340	0.422	0.714	1.080	0.595	1.307	0.807	1.569
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.25	0.51	0.317	0.304	0.221	0.219	0.156	0.161	0.485	0.415	1.092	0.750
B-splines	4	1	0.288	0.368	0.212	0.261	0.285	0.185	0.471	0.520	0.740	1.574
Partitioning	4	1	0.325	0.298	0.251	0.218	0.420	0.157	0.486	0.415	0.504	0.791
Feasible Estimation												
Local Polynomial	0.31	0.26	0.214	0.277	0.151	0.197	0.132	0.173	0.373	0.424	0.348	0.406
B-splines	3	3	0.281	0.424	0.205	0.299	0.268	0.254	0.458	0.682	0.710	2.063
Partitioning	3	3	0.311	0.495	0.235	0.355	0.396	0.886	0.466	1.114	0.479	1.404
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.549	0.492	0.370	0.368	0.169	0.229	0.430	0.520	0.931	1.182
B-splines	9	9	0.647	0.613	0.454	0.454	0.176	0.327	0.557	0.820	1.163	4.245
Partitioning	9	9	0.697	0.868	0.523	0.660	0.290	0.588	0.678	1.179	0.832	2.671
Feasible Estimation												
Local Polynomial	0.25	0.27	0.573	0.506	0.431	0.387	0.303	0.311	0.398	0.475	1.125	1.075
B-splines	4	3	0.621	0.581	0.410	0.436	0.354	0.349	0.613	0.735	0.875	2.269
Partitioning	4	3	0.638	0.613	0.442	0.462	0.480	1.010	0.650	1.245	0.660	1.546
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.26	0.9	0.314	0.254	0.228	0.192	0.225	0.175	0.454	0.357	1.003	0.498
B-splines	4	1	0.274	0.369	0.204	0.263	0.304	0.199	0.443	0.527	0.740	1.579
Partitioning	4	1	0.312	0.298	0.243	0.220	0.440	0.177	0.451	0.409	0.497	0.805
Feasible Estimation												
Local Polynomial	0.27	0.26	0.201	0.248	0.156	0.177	0.214	0.163	0.333	0.390	0.333	0.390
B-splines	4	2	0.276	0.414	0.206	0.292	0.307	0.244	0.447	0.648	0.740	2.011
Partitioning	4	2	0.313	0.461	0.244	0.327	0.442	0.784	0.454	0.995	0.502	1.307
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.161	0.243	0.123	0.183	0.087	0.147	0.193	0.343	0.273	0.487
B-splines	1	1	0.179	0.357	0.133	0.252	0.087	0.171	0.189	0.499	0.389	1.577
Partitioning	1	1	0.153	0.283	0.118	0.207	0.086	0.147	0.189	0.374	0.244	0.789
Feasible Estimation												
Local Polynomial	0.33	0.27	0.384	0.424	0.316	0.341	0.105	0.163	0.324	0.395	0.710	0.741
B-splines	3	2	0.247	0.388	0.179	0.272	0.231	0.207	0.395	0.578	0.667	1.829
Partitioning	3	2	0.269	0.397	0.200	0.275	0.349	0.632	0.392	0.766	0.441	1.133
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.300	0.432	0.206	0.308	0.954	0.929	0.512	0.613	0.825	1.174
B-splines	9	9	0.386	0.556	0.286	0.400	0.952	0.941	0.542	0.833	1.023	4.005
Partitioning	9	9	0.485	0.858	0.375	0.650	0.955	0.965	0.672	1.216	0.722	2.646
Feasible Estimation												
Local Polynomial	0.31	0.26	0.228	0.292	0.147	0.200	0.972	0.982	0.483	0.534	0.324	0.390
B-splines	3	3	0.300	0.443	0.213	0.313	0.947	0.945	0.543	0.747	0.705	2.058
Partitioning	3	3	0.326	0.498	0.241	0.355	0.951	1.135	0.512	1.117	0.477	1.355
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.45	0.9	0.216	0.244	0.156	0.184	0.100	0.150	0.280	0.343	0.573	0.488
B-splines	1	1	0.183	0.357	0.136	0.253	0.092	0.171	0.191	0.499	0.388	1.576
Partitioning	1	1	0.158	0.283	0.122	0.208	0.091	0.150	0.191	0.374	0.245	0.792
Feasible Estimation												
Local Polynomial	0.31	0.26	0.171	0.250	0.123	0.178	0.108	0.163	0.323	0.390	0.323	0.389
B-splines	3	3	0.261	0.416	0.191	0.292	0.258	0.244	0.424	0.654	0.705	2.041
Partitioning	3	3	0.293	0.476	0.223	0.337	0.394	0.825	0.424	1.059	0.474	1.339

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.43: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.17	0.23	0.161	0.191	0.124	0.148	0.167	0.184	0.157	0.193	0.177	0.220
B-splines	9	4	0.177	0.173	0.137	0.134	0.183	0.232	0.109	0.166	0.165	0.166
Partitioning	9	4	0.184	0.205	0.142	0.156	0.184	0.532	0.127	0.327	0.154	0.168
Feasible Estimation												
Local Polynomial	0.28	0.27	0.199	0.201	0.146	0.155	0.084	0.149	0.096	0.113	0.084	0.096
B-splines	4	1	0.166	0.177	0.124	0.138	0.384	0.243	0.158	0.125	0.141	0.162
Partitioning	4	1	0.177	0.175	0.133	0.134	0.306	0.213	0.248	0.168	0.138	0.159
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.206	0.233	0.161	0.180	0.202	0.213	0.196	0.273	0.182	0.323
B-splines	16	4	0.195	0.174	0.152	0.134	0.281	0.148	0.204	0.157	0.181	0.169
Partitioning	16	4	0.253	0.218	0.196	0.166	0.542	0.658	0.318	0.333	0.179	0.170
Feasible Estimation												
Local Polynomial	0.32	0.23	0.487	0.489	0.390	0.387	0.703	0.747	0.602	0.604	0.277	0.285
B-splines	4	4	0.344	0.174	0.276	0.134	0.202	0.148	0.159	0.157	0.228	0.169
Partitioning	4	4	0.373	0.218	0.304	0.166	0.850	0.658	0.278	0.333	0.309	0.170
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.15	0.37	0.168	0.156	0.129	0.120	0.141	0.106	0.168	0.133	0.186	0.174
B-splines	16	4	0.169	0.160	0.130	0.122	0.210	0.147	0.204	0.157	0.179	0.167
Partitioning	16	4	0.229	0.205	0.178	0.156	0.330	0.532	0.277	0.327	0.181	0.169
Feasible Estimation												
Local Polynomial	0.34	0.28	0.259	0.265	0.164	0.172	0.114	0.121	0.232	0.240	0.179	0.189
B-splines	4	3	0.187	0.167	0.145	0.129	0.205	0.148	0.140	0.146	0.140	0.165
Partitioning	4	3	0.200	0.196	0.155	0.149	0.199	0.427	0.243	0.280	0.128	0.179
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.666	0.318	0.479	0.242	0.140	0.175	0.306	0.151	0.197	0.197
B-splines	9	9	0.690	0.332	0.553	0.258	0.167	0.240	0.207	0.163	0.199	0.182
Partitioning	9	9	0.577	0.332	0.445	0.253	0.163	0.189	0.231	0.232	0.187	0.223
Feasible Estimation												
Local Polynomial	0.19	0.27	0.610	0.530	0.509	0.438	0.394	0.394	0.667	0.363	0.588	0.902
B-splines	4	1	0.699	0.409	0.559	0.322	0.247	0.387	0.199	0.188	0.196	0.185
Partitioning	4	1	0.599	0.360	0.460	0.274	0.269	0.582	0.225	0.342	0.183	0.173
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.145	0.123	0.112	0.099	0.156	0.170	0.143	0.085	0.174	0.108
B-splines	4	1	0.107	0.169	0.084	0.135	0.168	0.186	0.135	0.119	0.137	0.166
Partitioning	4	1	0.119	0.150	0.093	0.122	0.218	0.179	0.156	0.103	0.122	0.145
Feasible Estimation												
Local Polynomial	0.23	0.29	0.097	0.088	0.077	0.068	0.173	0.078	0.075	0.089	0.075	0.089
B-splines	4	1	0.107	0.169	0.084	0.134	0.168	0.185	0.135	0.119	0.137	0.166
Partitioning	4	1	0.119	0.153	0.093	0.123	0.218	0.204	0.156	0.115	0.122	0.147
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.060	0.090	0.046	0.071	0.033	0.066	0.057	0.084	0.083	0.107
B-splines	1	1	0.063	0.127	0.048	0.097	0.033	0.081	0.055	0.117	0.094	0.162
Partitioning	1	1	0.055	0.101	0.043	0.078	0.033	0.066	0.055	0.099	0.072	0.142
Feasible Estimation												
Local Polynomial	0.41	0.31	0.459	0.462	0.404	0.405	0.045	0.075	0.078	0.096	0.646	0.652
B-splines	1	1	0.066	0.127	0.050	0.097	0.042	0.081	0.065	0.117	0.097	0.162
Partitioning	1	1	0.060	0.101	0.045	0.078	0.056	0.066	0.065	0.099	0.075	0.142
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.155	0.198	0.091	0.139	0.960	0.924	0.356	0.353	0.151	0.188
B-splines	9	9	0.177	0.225	0.115	0.161	0.967	0.941	0.359	0.360	0.165	0.172
Partitioning	9	9	0.204	0.320	0.145	0.241	0.949	0.891	0.356	0.384	0.153	0.218
Feasible Estimation												
Local Polynomial	0.34	0.28	0.137	0.152	0.064	0.085	0.984	0.986	0.360	0.362	0.074	0.090
B-splines	2	1	0.145	0.177	0.077	0.115	0.968	0.959	0.364	0.359	0.114	0.162
Partitioning	2	1	0.146	0.161	0.077	0.099	0.953	0.958	0.358	0.354	0.095	0.144
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.29	0.9	0.104	0.096	0.079	0.075	0.071	0.080	0.111	0.086	0.159	0.108
B-splines	4	1	0.097	0.128	0.075	0.098	0.132	0.084	0.134	0.117	0.135	0.162
Partitioning	4	1	0.114	0.103	0.089	0.080	0.192	0.082	0.155	0.101	0.122	0.145
Feasible Estimation												
Local Polynomial	0.34	0.28	0.079	0.102	0.059	0.078	0.047	0.072	0.073	0.090	0.073	0.090
B-splines	3	1	0.093	0.130	0.070	0.099	0.101	0.088	0.108	0.118	0.121	0.162
Partitioning	3	1	0.102	0.113	0.077	0.085	0.153	0.152	0.126	0.121	0.106	0.147

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.44: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.16	0.23	0.166	0.190	0.126	0.144	0.150	0.161	0.178	0.196	0.253	0.268
B-splines	9	4	0.186	0.175	0.146	0.134	0.210	0.206	0.133	0.184	0.209	0.240
Partitioning	9	4	0.183	0.205	0.143	0.156	0.165	0.475	0.151	0.353	0.185	0.231
Feasible Estimation												
Local Polynomial	0.27	0.27	0.215	0.213	0.165	0.170	0.059	0.126	0.101	0.115	0.098	0.105
B-splines	4	1	0.173	0.187	0.132	0.146	0.368	0.236	0.199	0.153	0.169	0.227
Partitioning	4	1	0.180	0.184	0.138	0.143	0.324	0.235	0.280	0.200	0.154	0.199
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.203	0.227	0.156	0.171	0.177	0.174	0.210	0.262	0.259	0.401
B-splines	16	4	0.207	0.171	0.160	0.129	0.277	0.128	0.255	0.182	0.245	0.242
Partitioning	16	4	0.251	0.216	0.195	0.164	0.433	0.567	0.381	0.361	0.215	0.231
Feasible Estimation												
Local Polynomial	0.29	0.23	0.505	0.508	0.411	0.410	0.675	0.704	0.571	0.572	0.313	0.320
B-splines	4	4	0.328	0.171	0.258	0.129	0.173	0.128	0.164	0.182	0.335	0.242
Partitioning	4	4	0.362	0.216	0.292	0.164	0.721	0.567	0.301	0.361	0.427	0.231
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.16	0.39	0.160	0.148	0.120	0.110	0.109	0.091	0.177	0.138	0.258	0.203
B-splines	9	4	0.176	0.159	0.136	0.118	0.117	0.128	0.148	0.181	0.212	0.240
Partitioning	9	4	0.195	0.203	0.150	0.154	0.104	0.487	0.159	0.356	0.192	0.235
Feasible Estimation												
Local Polynomial	0.34	0.28	0.210	0.217	0.133	0.140	0.104	0.103	0.214	0.214	0.164	0.166
B-splines	4	3	0.172	0.164	0.128	0.124	0.162	0.130	0.164	0.171	0.169	0.236
Partitioning	4	3	0.182	0.190	0.137	0.142	0.171	0.388	0.241	0.301	0.136	0.230
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.613	0.322	0.425	0.246	0.138	0.158	0.190	0.155	0.223	0.224
B-splines	9	9	0.676	0.347	0.506	0.275	0.137	0.255	0.155	0.193	0.296	0.276
Partitioning	9	9	0.619	0.349	0.464	0.269	0.134	0.194	0.185	0.252	0.234	0.280
Feasible Estimation												
Local Polynomial	0.19	0.27	0.580	0.506	0.471	0.403	0.345	0.344	0.571	0.325	0.690	0.945
B-splines	4	2	0.686	0.399	0.510	0.322	0.245	0.343	0.179	0.205	0.289	0.257
Partitioning	4	2	0.641	0.335	0.474	0.261	0.271	0.570	0.205	0.367	0.238	0.237
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.145	0.120	0.110	0.097	0.139	0.149	0.158	0.098	0.235	0.125
B-splines	4	1	0.107	0.166	0.082	0.130	0.163	0.162	0.146	0.146	0.168	0.235
Partitioning	4	1	0.118	0.147	0.092	0.118	0.211	0.156	0.149	0.126	0.136	0.182
Feasible Estimation												
Local Polynomial	0.23	0.29	0.099	0.087	0.079	0.066	0.161	0.067	0.088	0.096	0.088	0.096
B-splines	4	1	0.107	0.166	0.082	0.130	0.163	0.162	0.146	0.149	0.168	0.234
Partitioning	4	1	0.118	0.151	0.092	0.120	0.211	0.183	0.149	0.137	0.136	0.186
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.059	0.089	0.046	0.069	0.033	0.061	0.060	0.094	0.089	0.122
B-splines	1	1	0.064	0.127	0.049	0.094	0.033	0.072	0.058	0.129	0.109	0.226
Partitioning	1	1	0.055	0.100	0.043	0.076	0.033	0.060	0.057	0.107	0.077	0.171
Feasible Estimation												
Local Polynomial	0.4	0.31	0.419	0.423	0.362	0.364	0.042	0.065	0.086	0.103	0.646	0.651
B-splines	1	1	0.066	0.127	0.050	0.094	0.041	0.072	0.067	0.129	0.115	0.226
Partitioning	1	1	0.060	0.100	0.046	0.076	0.050	0.060	0.067	0.107	0.083	0.171
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.164	0.202	0.093	0.137	0.957	0.918	0.359	0.355	0.183	0.215
B-splines	9	9	0.185	0.230	0.118	0.161	0.959	0.930	0.364	0.371	0.209	0.254
Partitioning	9	9	0.211	0.323	0.148	0.243	0.935	0.856	0.367	0.387	0.179	0.269
Feasible Estimation												
Local Polynomial	0.34	0.28	0.149	0.162	0.068	0.087	0.978	0.979	0.361	0.363	0.084	0.097
B-splines	2	1	0.157	0.186	0.085	0.118	0.955	0.952	0.367	0.369	0.145	0.226
Partitioning	2	1	0.158	0.173	0.087	0.105	0.937	0.940	0.348	0.370	0.112	0.178
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.3	0.9	0.103	0.094	0.076	0.072	0.061	0.070	0.119	0.094	0.197	0.122
B-splines	4	1	0.096	0.127	0.073	0.095	0.117	0.074	0.146	0.129	0.167	0.226
Partitioning	4	1	0.112	0.102	0.087	0.078	0.171	0.072	0.149	0.109	0.135	0.176
Feasible Estimation												
Local Polynomial	0.34	0.28	0.074	0.097	0.055	0.073	0.043	0.063	0.083	0.096	0.083	0.097
B-splines	3	1	0.091	0.131	0.069	0.097	0.093	0.079	0.121	0.138	0.150	0.225
Partitioning	3	1	0.100	0.117	0.076	0.086	0.144	0.159	0.124	0.157	0.118	0.183

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.45: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.15	0.22	0.174	0.187	0.125	0.134	0.127	0.130	0.237	0.229	0.739	0.554
B-splines	9	4	0.199	0.176	0.158	0.128	0.223	0.159	0.240	0.235	0.355	0.633
Partitioning	9	4	0.187	0.206	0.146	0.154	0.133	0.428	0.208	0.452	0.281	0.498
Feasible Estimation												
Local Polynomial	0.25	0.26	0.227	0.221	0.183	0.180	0.043	0.098	0.124	0.144	0.136	0.145
B-splines	4	2	0.180	0.194	0.141	0.150	0.314	0.212	0.309	0.240	0.257	0.560
Partitioning	4	2	0.186	0.199	0.144	0.154	0.335	0.318	0.375	0.324	0.195	0.391
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.201	0.216	0.145	0.154	0.146	0.132	0.263	0.293	0.637	1.093
B-splines	16	4	0.218	0.166	0.166	0.118	0.261	0.102	0.329	0.230	0.507	0.634
Partitioning	16	4	0.254	0.212	0.197	0.158	0.356	0.468	0.422	0.468	0.441	0.509
Feasible Estimation												
Local Polynomial	0.22	0.22	0.513	0.516	0.420	0.422	0.606	0.613	0.510	0.514	0.392	0.400
B-splines	8	4	0.289	0.166	0.224	0.118	0.284	0.102	0.204	0.230	0.589	0.634
Partitioning	8	4	0.269	0.212	0.208	0.158	0.247	0.468	0.216	0.468	0.608	0.509
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.17	0.42	0.147	0.130	0.102	0.092	0.079	0.073	0.211	0.162	0.537	0.344
B-splines	9	4	0.160	0.158	0.119	0.111	0.090	0.103	0.213	0.229	0.365	0.633
Partitioning	9	4	0.184	0.202	0.141	0.150	0.100	0.419	0.215	0.460	0.258	0.508
Feasible Estimation												
Local Polynomial	0.34	0.28	0.146	0.155	0.098	0.104	0.089	0.083	0.207	0.197	0.164	0.164
B-splines	3	3	0.140	0.157	0.098	0.111	0.110	0.102	0.227	0.221	0.243	0.599
Partitioning	3	3	0.147	0.184	0.106	0.134	0.142	0.349	0.251	0.384	0.187	0.464
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.496	0.313	0.320	0.239	0.131	0.133	0.227	0.172	0.456	0.380
B-splines	9	9	0.564	0.359	0.369	0.284	0.075	0.235	0.384	0.275	0.557	1.112
Partitioning	9	9	0.557	0.358	0.379	0.273	0.110	0.198	0.403	0.342	0.457	0.671
Feasible Estimation												
Local Polynomial	0.21	0.27	0.512	0.450	0.390	0.342	0.273	0.263	0.359	0.271	0.927	1.024
B-splines	4	4	0.570	0.398	0.360	0.320	0.206	0.274	0.444	0.241	0.529	0.691
Partitioning	4	4	0.568	0.327	0.366	0.256	0.236	0.509	0.469	0.459	0.461	0.521
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.146	0.112	0.106	0.088	0.120	0.114	0.199	0.147	0.498	0.194
B-splines	4	1	0.107	0.157	0.082	0.117	0.160	0.122	0.153	0.223	0.253	0.528
Partitioning	4	1	0.119	0.137	0.093	0.105	0.206	0.119	0.166	0.197	0.183	0.313
Feasible Estimation												
Local Polynomial	0.22	0.29	0.100	0.087	0.082	0.062	0.145	0.060	0.123	0.135	0.123	0.135
B-splines	4	1	0.107	0.158	0.082	0.117	0.160	0.121	0.153	0.226	0.253	0.547
Partitioning	4	1	0.119	0.147	0.093	0.111	0.206	0.185	0.166	0.233	0.183	0.335
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.058	0.086	0.045	0.064	0.033	0.052	0.071	0.122	0.101	0.179
B-splines	1	1	0.063	0.126	0.047	0.088	0.033	0.060	0.067	0.164	0.137	0.494
Partitioning	1	1	0.055	0.099	0.043	0.072	0.033	0.052	0.067	0.130	0.088	0.278
Feasible Estimation												
Local Polynomial	0.4	0.3	0.353	0.358	0.296	0.299	0.038	0.058	0.115	0.140	0.652	0.658
B-splines	1	1	0.070	0.126	0.051	0.088	0.050	0.060	0.087	0.164	0.172	0.496
Partitioning	1	1	0.068	0.099	0.050	0.073	0.071	0.060	0.092	0.130	0.112	0.279
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.177	0.205	0.097	0.131	0.950	0.910	0.363	0.355	0.294	0.371
B-splines	9	9	0.198	0.241	0.125	0.163	0.943	0.915	0.368	0.399	0.349	0.849
Partitioning	9	9	0.220	0.327	0.154	0.245	0.910	0.806	0.370	0.436	0.254	0.662
Feasible Estimation												
Local Polynomial	0.33	0.27	0.166	0.178	0.078	0.094	0.969	0.969	0.362	0.370	0.116	0.137
B-splines	3	2	0.174	0.204	0.099	0.127	0.928	0.930	0.362	0.386	0.232	0.551
Partitioning	3	2	0.177	0.199	0.105	0.125	0.898	0.877	0.332	0.388	0.171	0.367
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.32	0.9	0.096	0.089	0.067	0.067	0.048	0.058	0.134	0.123	0.309	0.179
B-splines	4	1	0.095	0.126	0.071	0.089	0.101	0.062	0.151	0.165	0.252	0.494
Partitioning	4	1	0.111	0.101	0.086	0.074	0.150	0.058	0.164	0.130	0.183	0.288
Feasible Estimation												
Local Polynomial	0.34	0.28	0.067	0.092	0.049	0.066	0.042	0.058	0.113	0.135	0.114	0.136
B-splines	3	2	0.091	0.136	0.067	0.095	0.088	0.074	0.132	0.184	0.233	0.546
Partitioning	3	2	0.101	0.135	0.076	0.093	0.131	0.207	0.140	0.232	0.166	0.359

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.46: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.21	0.27	0.267	0.345	0.205	0.266	0.240	0.305	0.271	0.327	0.339	0.370
B-splines	4	4	0.229	0.325	0.177	0.250	0.462	0.342	0.287	0.318	0.270	0.333
Partitioning	4	4	0.256	0.404	0.198	0.307	0.438	1.055	0.381	0.651	0.251	0.336
Feasible Estimation												
Local Polynomial	0.29	0.26	0.225	0.252	0.171	0.198	0.119	0.200	0.159	0.193	0.153	0.184
B-splines	3	2	0.233	0.299	0.180	0.230	0.426	0.303	0.273	0.264	0.269	0.326
Partitioning	3	2	0.255	0.310	0.196	0.231	0.411	0.598	0.353	0.402	0.247	0.307
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.15	0.21	0.343	0.412	0.266	0.319	0.314	0.361	0.349	0.440	0.361	0.516
B-splines	9	4	0.444	0.326	0.362	0.250	0.530	0.292	0.460	0.313	0.340	0.334
Partitioning	9	4	0.396	0.410	0.312	0.312	0.365	1.128	0.337	0.654	0.310	0.337
Feasible Estimation												
Local Polynomial	0.31	0.24	0.498	0.512	0.399	0.407	0.709	0.757	0.616	0.626	0.304	0.323
B-splines	4	4	0.383	0.327	0.306	0.251	0.304	0.294	0.280	0.312	0.327	0.334
Partitioning	4	4	0.419	0.411	0.337	0.313	0.899	1.126	0.384	0.652	0.375	0.338
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.19	0.43	0.279	0.271	0.213	0.208	0.219	0.176	0.286	0.223	0.352	0.300
B-splines	9	4	0.284	0.318	0.220	0.244	0.175	0.292	0.222	0.313	0.331	0.333
Partitioning	9	4	0.348	0.403	0.271	0.307	0.199	1.051	0.253	0.651	0.316	0.337
Feasible Estimation												
Local Polynomial	0.32	0.27	0.279	0.305	0.186	0.214	0.142	0.178	0.263	0.288	0.218	0.246
B-splines	4	3	0.250	0.304	0.192	0.232	0.296	0.256	0.276	0.280	0.267	0.328
Partitioning	4	3	0.274	0.343	0.212	0.255	0.352	0.771	0.358	0.490	0.240	0.330
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.686	0.422	0.501	0.332	0.175	0.259	0.351	0.295	0.325	0.382
B-splines	9	9	0.724	0.468	0.580	0.365	0.201	0.316	0.277	0.294	0.348	0.349
Partitioning	9	9	0.644	0.617	0.505	0.471	0.234	0.377	0.315	0.459	0.323	0.440
Feasible Estimation												
Local Polynomial	0.2	0.27	0.648	0.554	0.532	0.452	0.417	0.423	0.669	0.393	0.613	0.921
B-splines	4	2	0.803	0.562	0.621	0.443	0.629	0.424	0.281	0.311	0.298	0.343
Partitioning	4	2	0.793	0.559	0.611	0.437	0.699	0.920	0.336	0.553	0.272	0.325
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.24	0.9	0.243	0.199	0.187	0.158	0.243	0.207	0.246	0.168	0.325	0.215
B-splines	4	1	0.198	0.278	0.153	0.215	0.283	0.234	0.268	0.234	0.271	0.326
Partitioning	4	1	0.226	0.230	0.176	0.181	0.375	0.214	0.311	0.200	0.244	0.285
Feasible Estimation												
Local Polynomial	0.26	0.27	0.152	0.176	0.120	0.136	0.223	0.151	0.148	0.181	0.148	0.181
B-splines	4	2	0.198	0.291	0.153	0.223	0.283	0.254	0.268	0.258	0.271	0.326
Partitioning	4	2	0.226	0.290	0.176	0.216	0.375	0.555	0.311	0.368	0.244	0.301
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.119	0.181	0.092	0.141	0.066	0.132	0.113	0.168	0.165	0.215
B-splines	1	1	0.127	0.255	0.097	0.194	0.066	0.161	0.110	0.233	0.188	0.324
Partitioning	1	1	0.110	0.202	0.086	0.156	0.066	0.133	0.110	0.199	0.144	0.284
Feasible Estimation												
Local Polynomial	0.35	0.28	0.470	0.486	0.410	0.418	0.093	0.148	0.149	0.181	0.660	0.671
B-splines	2	1	0.163	0.260	0.122	0.197	0.182	0.171	0.213	0.237	0.234	0.324
Partitioning	2	1	0.175	0.223	0.127	0.167	0.262	0.290	0.241	0.241	0.198	0.289
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.227	0.341	0.160	0.259	0.967	0.944	0.394	0.431	0.301	0.376
B-splines	9	9	0.283	0.399	0.209	0.302	0.976	0.964	0.401	0.433	0.330	0.344
Partitioning	9	9	0.351	0.611	0.270	0.466	0.964	0.950	0.412	0.549	0.306	0.437
Feasible Estimation												
Local Polynomial	0.32	0.27	0.172	0.215	0.107	0.150	0.989	0.997	0.381	0.393	0.147	0.181
B-splines	3	2	0.214	0.299	0.147	0.220	0.981	0.969	0.409	0.426	0.250	0.326
Partitioning	3	2	0.229	0.295	0.159	0.208	0.972	1.035	0.415	0.479	0.220	0.300
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.37	0.9	0.183	0.183	0.138	0.144	0.109	0.139	0.189	0.169	0.288	0.215
B-splines	1	1	0.138	0.255	0.106	0.194	0.069	0.162	0.112	0.233	0.188	0.324
Partitioning	1	1	0.123	0.203	0.096	0.157	0.069	0.141	0.112	0.200	0.144	0.285
Feasible Estimation												
Local Polynomial	0.32	0.27	0.130	0.183	0.100	0.142	0.095	0.148	0.147	0.181	0.147	0.182
B-splines	3	2	0.183	0.278	0.139	0.210	0.229	0.211	0.245	0.263	0.256	0.326
Partitioning	3	2	0.206	0.285	0.156	0.205	0.321	0.579	0.280	0.386	0.229	0.304

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.47: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1, 1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		Point Estimation RMSE (0.1,0.5)		Point Estimation RMSE (0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 2.1												
Infeasible Estimation												
Local Polynomial	0.2	0.26	0.275	0.342	0.207	0.258	0.217	0.265	0.307	0.339	0.460	0.450
B-splines	4	4	0.236	0.324	0.183	0.244	0.424	0.302	0.322	0.363	0.335	0.480
Partitioning	4	4	0.259	0.402	0.201	0.305	0.431	0.943	0.379	0.704	0.279	0.460
Feasible Estimation												
Local Polynomial	0.29	0.26	0.239	0.261	0.187	0.208	0.094	0.170	0.178	0.204	0.177	0.200
B-splines	4	2	0.240	0.304	0.187	0.232	0.403	0.274	0.308	0.311	0.329	0.463
Partitioning	4	2	0.259	0.318	0.201	0.238	0.414	0.574	0.365	0.481	0.277	0.394
Model 2.2												
Infeasible Estimation												
Local Polynomial	0.15	0.21	0.337	0.402	0.256	0.303	0.272	0.299	0.376	0.431	0.510	0.620
B-splines	9	4	0.418	0.322	0.339	0.241	0.444	0.254	0.397	0.362	0.457	0.481
Partitioning	9	4	0.390	0.408	0.306	0.309	0.299	0.993	0.327	0.707	0.398	0.459
Feasible Estimation												
Local Polynomial	0.3	0.24	0.516	0.529	0.419	0.427	0.677	0.715	0.592	0.600	0.343	0.360
B-splines	4	4	0.365	0.323	0.286	0.242	0.246	0.255	0.305	0.362	0.447	0.482
Partitioning	4	4	0.413	0.408	0.330	0.309	0.791	0.992	0.408	0.705	0.496	0.460
Model 2.3												
Infeasible Estimation												
Local Polynomial	0.2	0.44	0.268	0.255	0.198	0.191	0.172	0.153	0.305	0.238	0.464	0.350
B-splines	9	4	0.281	0.316	0.215	0.236	0.163	0.254	0.253	0.362	0.418	0.480
Partitioning	9	4	0.345	0.401	0.268	0.304	0.199	0.955	0.304	0.706	0.365	0.463
Feasible Estimation												
Local Polynomial	0.32	0.27	0.233	0.264	0.155	0.183	0.127	0.152	0.257	0.272	0.218	0.236
B-splines	4	3	0.235	0.300	0.176	0.224	0.246	0.219	0.301	0.321	0.329	0.470
Partitioning	4	3	0.257	0.336	0.197	0.246	0.316	0.684	0.353	0.548	0.266	0.427
Model 2.4												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.635	0.420	0.448	0.327	0.166	0.225	0.266	0.302	0.390	0.436
B-splines	9	9	0.711	0.477	0.538	0.370	0.174	0.314	0.257	0.346	0.470	0.520
Partitioning	9	9	0.681	0.625	0.526	0.479	0.216	0.387	0.320	0.494	0.391	0.544
Feasible Estimation												
Local Polynomial	0.21	0.27	0.618	0.531	0.495	0.421	0.365	0.367	0.567	0.364	0.732	0.967
B-splines	4	2	0.745	0.536	0.540	0.418	0.490	0.386	0.343	0.363	0.388	0.492
Partitioning	4	2	0.745	0.529	0.547	0.407	0.573	0.857	0.385	0.626	0.350	0.451
Model 2.5												
Infeasible Estimation												
Local Polynomial	0.24	0.9	0.244	0.195	0.184	0.153	0.216	0.184	0.271	0.189	0.419	0.245
B-splines	4	1	0.196	0.275	0.150	0.208	0.259	0.206	0.291	0.267	0.335	0.456
Partitioning	4	1	0.223	0.228	0.174	0.176	0.350	0.189	0.297	0.225	0.270	0.348
Feasible Estimation												
Local Polynomial	0.25	0.27	0.152	0.173	0.121	0.131	0.205	0.129	0.172	0.195	0.172	0.195
B-splines	4	2	0.196	0.288	0.150	0.216	0.259	0.215	0.291	0.301	0.335	0.462
Partitioning	4	2	0.223	0.287	0.174	0.211	0.350	0.475	0.297	0.419	0.270	0.377
Model 2.6												
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.118	0.178	0.091	0.137	0.066	0.121	0.120	0.187	0.179	0.243
B-splines	1	1	0.127	0.254	0.097	0.188	0.065	0.143	0.115	0.258	0.218	0.453
Partitioning	1	1	0.110	0.201	0.086	0.153	0.065	0.121	0.115	0.214	0.154	0.343
Feasible Estimation												
Local Polynomial	0.34	0.28	0.431	0.449	0.368	0.379	0.084	0.127	0.170	0.197	0.663	0.673
B-splines	2	1	0.164	0.261	0.122	0.193	0.169	0.155	0.225	0.271	0.292	0.451
Partitioning	2	1	0.175	0.230	0.128	0.168	0.244	0.303	0.226	0.313	0.224	0.356
Model 2.7												
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.232	0.337	0.159	0.249	0.963	0.931	0.404	0.437	0.366	0.431
B-splines	9	9	0.287	0.400	0.210	0.296	0.969	0.947	0.419	0.468	0.417	0.507
Partitioning	9	9	0.355	0.611	0.271	0.465	0.949	0.913	0.449	0.572	0.358	0.537
Feasible Estimation												
Local Polynomial	0.32	0.27	0.180	0.221	0.109	0.149	0.981	0.985	0.389	0.399	0.169	0.195
B-splines	3	2	0.222	0.305	0.153	0.220	0.955	0.959	0.437	0.461	0.312	0.462
Partitioning	3	2	0.236	0.306	0.167	0.214	0.937	0.996	0.414	0.541	0.250	0.376
Model 2.8												
Infeasible Estimation												
Local Polynomial	0.38	0.9	0.177	0.180	0.130	0.139	0.095	0.126	0.194	0.187	0.336	0.243
B-splines	1	1	0.136	0.254	0.104	0.188	0.070	0.144	0.119	0.258	0.219	0.453
Partitioning	1	1	0.121	0.202	0.094	0.154	0.070	0.126	0.118	0.215	0.155	0.346
Feasible Estimation												
Local Polynomial	0.32	0.27	0.125	0.179	0.095	0.135	0.086	0.128	0.169	0.195	0.169	0.196
B-splines	3	2	0.182	0.277	0.137	0.204	0.206	0.178	0.267	0.304	0.315	0.465
Partitioning	3	2	0.204	0.285	0.155	0.203	0.298	0.530	0.276	0.448	0.252	0.383

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.48: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 2$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2,2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5)		(0.1,0.5)		(0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation												
Local Polynomial	0.19	0.26	0.285	0.334	0.204	0.238	0.186	0.217	0.397	0.397	0.996	0.883
B-splines	9	4	0.295	0.325	0.230	0.231	0.246	0.240	0.340	0.461	0.701	1.266
Partitioning	9	4	0.340	0.404	0.264	0.301	0.215	0.832	0.379	0.900	0.521	0.987
Feasible Estimation												
Local Polynomial	0.28	0.26	0.252	0.268	0.203	0.214	0.083	0.141	0.239	0.277	0.248	0.279
B-splines	4	3	0.245	0.316	0.190	0.230	0.354	0.247	0.394	0.420	0.507	1.161
Partitioning	4	3	0.265	0.356	0.207	0.262	0.415	0.645	0.460	0.693	0.370	0.856
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.16	0.21	0.332	0.383	0.238	0.272	0.227	0.228	0.467	0.481	1.373	1.300
B-splines	9	4	0.364	0.320	0.282	0.225	0.321	0.204	0.321	0.458	0.842	1.267
Partitioning	9	4	0.382	0.407	0.297	0.303	0.237	0.854	0.363	0.911	0.742	0.989
Feasible Estimation												
Local Polynomial	0.25	0.23	0.524	0.537	0.428	0.438	0.610	0.624	0.557	0.570	0.435	0.460
B-splines	4	4	0.322	0.320	0.241	0.225	0.214	0.206	0.406	0.458	0.746	1.262
Partitioning	4	4	0.386	0.407	0.299	0.303	0.633	0.853	0.480	0.909	0.742	0.991
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.22	0.48	0.245	0.226	0.169	0.162	0.124	0.123	0.348	0.290	0.842	0.584
B-splines	4	4	0.216	0.316	0.158	0.221	0.209	0.205	0.339	0.458	0.509	1.267
Partitioning	4	4	0.242	0.402	0.185	0.299	0.296	0.831	0.379	0.906	0.375	0.994
Feasible Estimation												
Local Polynomial	0.31	0.27	0.178	0.216	0.124	0.151	0.111	0.131	0.282	0.306	0.254	0.286
B-splines	3	3	0.211	0.302	0.153	0.211	0.197	0.188	0.336	0.419	0.487	1.194
Partitioning	3	3	0.233	0.355	0.175	0.254	0.279	0.687	0.362	0.727	0.357	0.894
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.521	0.399	0.345	0.303	0.149	0.183	0.319	0.338	0.690	0.748
B-splines	9	9	0.605	0.487	0.412	0.368	0.126	0.280	0.452	0.481	0.823	1.852
Partitioning	9	9	0.625	0.631	0.454	0.479	0.202	0.395	0.514	0.677	0.638	1.330
Feasible Estimation												
Local Polynomial	0.23	0.27	0.545	0.477	0.412	0.363	0.292	0.284	0.351	0.355	1.036	1.057
B-splines	4	3	0.596	0.492	0.382	0.377	0.301	0.311	0.521	0.476	0.670	1.288
Partitioning	4	3	0.604	0.486	0.403	0.369	0.381	0.837	0.567	0.859	0.558	0.975
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.23	0.9	0.243	0.186	0.175	0.141	0.183	0.145	0.329	0.256	0.789	0.363
B-splines	4	1	0.195	0.268	0.146	0.192	0.237	0.160	0.303	0.361	0.504	1.006
Partitioning	4	1	0.223	0.219	0.173	0.163	0.331	0.149	0.329	0.298	0.365	0.573
Feasible Estimation												
Local Polynomial	0.25	0.27	0.154	0.174	0.121	0.125	0.183	0.118	0.240	0.272	0.240	0.272
B-splines	4	2	0.195	0.295	0.146	0.208	0.237	0.181	0.303	0.411	0.504	1.141
Partitioning	4	2	0.223	0.325	0.173	0.231	0.331	0.576	0.329	0.632	0.365	0.822
					Model 2.6							
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.116	0.172	0.089	0.129	0.065	0.104	0.142	0.244	0.202	0.357
B-splines	1	1	0.126	0.251	0.094	0.177	0.065	0.119	0.134	0.327	0.274	0.988
Partitioning	1	1	0.110	0.198	0.086	0.145	0.065	0.103	0.134	0.259	0.175	0.557
Feasible Estimation												
Local Polynomial	0.34	0.28	0.367	0.388	0.305	0.319	0.077	0.117	0.230	0.273	0.687	0.703
B-splines	3	2	0.172	0.270	0.125	0.188	0.166	0.145	0.258	0.368	0.456	1.090
Partitioning	3	2	0.189	0.267	0.139	0.184	0.244	0.416	0.271	0.467	0.321	0.708
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.238	0.323	0.156	0.226	0.952	0.916	0.424	0.455	0.588	0.743
B-splines	9	9	0.295	0.408	0.213	0.289	0.948	0.926	0.442	0.561	0.698	1.693
Partitioning	9	9	0.359	0.614	0.274	0.463	0.924	0.871	0.483	0.726	0.509	1.325
Feasible Estimation												
Local Polynomial	0.31	0.26	0.196	0.234	0.116	0.151	0.970	0.971	0.411	0.435	0.234	0.273
B-splines	3	3	0.235	0.328	0.160	0.227	0.929	0.929	0.436	0.521	0.491	1.158
Partitioning	3	3	0.252	0.358	0.180	0.252	0.903	0.963	0.413	0.720	0.355	0.846
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.4	0.9	0.165	0.173	0.118	0.130	0.078	0.107	0.221	0.245	0.475	0.357
B-splines	1	1	0.132	0.252	0.099	0.177	0.072	0.120	0.138	0.328	0.276	0.988
Partitioning	1	1	0.117	0.199	0.091	0.145	0.072	0.107	0.139	0.259	0.177	0.562
Feasible Estimation												
Local Polynomial	0.31	0.27	0.123	0.177	0.090	0.127	0.080	0.118	0.231	0.273	0.232	0.273
B-splines	3	3	0.183	0.291	0.134	0.202	0.186	0.169	0.282	0.401	0.487	1.155
Partitioning	3	3	0.207	0.333	0.157	0.233	0.277	0.615	0.299	0.668	0.349	0.846

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

C.3 Trivariate Simulations

C.3.1 Uniform Cell Boundaries

Table C.49: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.462	0.468	0.405	0.406	0.051	0.099	0.090	0.133	0.131	0.164	0.658	0.663
B-splines	1	1	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
Feasible Estimation														
Local Polynomial	0.45	0.31	0.648	0.653	0.529	0.533	0.062	0.115	0.118	0.155	0.651	0.659	1.285	1.289
B-splines	1	1	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.401	0.426	0.314	0.333	0.470	0.508	0.426	0.482	0.416	0.475	0.344	0.371
B-splines	27	8	0.398	0.413	0.314	0.321	0.407	0.509	0.361	0.456	0.429	0.422	0.306	0.328
Partitioning	27	8	0.493	0.576	0.383	0.444	0.585	4.445	1.113	2.740	0.481	1.557	0.444	0.746
Feasible Estimation														
Local Polynomial	0.38	0.28	0.373	0.383	0.286	0.296	0.360	0.361	0.293	0.308	0.308	0.315	0.275	0.290
B-splines	1	1	0.376	0.398	0.289	0.312	0.375	0.413	0.271	0.366	0.359	0.425	0.262	0.311
Partitioning	1	1	0.377	0.398	0.290	0.312	0.395	0.400	0.299	0.553	0.346	0.335	0.270	0.354
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.25	0.32	0.188	0.253	0.138	0.191	0.120	0.166	0.178	0.221	0.231	0.287	0.231	0.287
B-splines	8	1	0.167	0.204	0.122	0.152	0.207	0.116	0.201	0.166	0.193	0.236	0.193	0.236
Partitioning	8	1	0.258	0.207	0.201	0.159	0.493	0.106	0.405	0.175	0.347	0.241	0.286	0.357
Feasible Estimation														
Local Polynomial	0.39	0.28	0.142	0.169	0.087	0.117	0.061	0.100	0.101	0.125	0.101	0.125	0.101	0.125
B-splines	1	1	0.140	0.204	0.094	0.152	0.058	0.116	0.077	0.166	0.137	0.236	0.137	0.236
Partitioning	1	1	0.142	0.207	0.097	0.159	0.077	0.106	0.081	0.175	0.120	0.241	0.163	0.357
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.07	0.9	1.791	2.044	0.814	0.847	0.644	0.180	0.513	0.357	1.378	0.678	4.044	3.830
B-splines	216	27	1.955	1.903	0.905	0.880	0.633	0.193	0.602	0.465	1.614	1.346	4.138	4.021
Partitioning	216	27	1.443	1.113	0.598	0.554	0.243	0.024	0.576	0.523	1.278	1.796	3.388	4.360
Feasible Estimation														
Local Polynomial	0.29	0.26	2.077	2.061	0.820	0.839	0.070	0.107	0.475	0.336	0.475	0.336	3.792	3.787
B-splines	8	1	2.061	2.005	0.882	0.880	0.282	0.157	0.426	0.446	0.838	1.344	3.864	4.025
Partitioning	8	1	1.598	1.743	1.015	1.018	2.690	1.147	0.641	0.523	1.529	0.801	1.456	2.308
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.819	0.598	0.639	0.477	1.138	1.281	1.305	1.141	0.617	0.792	0.642	0.509
B-splines	27	27	0.878	0.617	0.699	0.493	0.843	1.428	1.158	1.313	0.794	0.684	0.510	0.606
Partitioning	27	27	0.764	1.276	0.604	0.991	0.936	0.383	1.394	1.109	0.560	3.010	0.533	3.962
Feasible Estimation														
Local Polynomial	0.27	0.28	0.756	0.573	0.592	0.460	1.205	1.219	1.359	1.155	0.476	0.681	0.690	0.493
B-splines	8	1	0.936	0.822	0.734	0.661	0.562	0.716	1.175	1.166	0.754	0.782	0.499	0.545
Partitioning	8	1	0.901	0.801	0.706	0.641	1.016	0.870	1.449	1.322	0.814	0.588	0.632	0.604

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.50: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.421	0.429	0.362	0.366	0.047	0.094	0.090	0.142	0.130	0.191	0.654	0.667
B-splines	1	1	0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
Partitioning	1	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
Feasible Estimation														
Local Polynomial	0.44	0.3	0.591	0.596	0.482	0.486	0.057	0.100	0.126	0.164	0.655	0.663	1.289	1.292
B-splines	1	1	0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
Partitioning	1	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.407	0.430	0.317	0.335	0.435	0.462	0.395	0.420	0.462	0.487	0.472	0.487
B-splines	27	8	0.403	0.421	0.317	0.327	0.401	0.463	0.346	0.415	0.437	0.483	0.406	0.491
Partitioning	27	8	0.491	0.575	0.383	0.442	0.341	3.287	0.430	2.566	0.556	1.861	0.630	1.296
Feasible Estimation														
Local Polynomial	0.37	0.27	0.374	0.384	0.286	0.296	0.374	0.380	0.312	0.327	0.300	0.308	0.294	0.309
B-splines	2	1	0.377	0.402	0.289	0.314	0.382	0.408	0.304	0.351	0.333	0.450	0.308	0.433
Partitioning	2	1	0.379	0.404	0.291	0.315	0.400	0.411	0.338	0.481	0.330	0.368	0.345	0.484
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.28	0.34	0.168	0.231	0.120	0.171	0.088	0.132	0.164	0.197	0.282	0.323	0.282	0.323
B-splines	8	1	0.156	0.195	0.114	0.142	0.182	0.105	0.203	0.178	0.236	0.331	0.236	0.331
Partitioning	8	1	0.258	0.206	0.201	0.155	0.443	0.105	0.396	0.184	0.367	0.284	0.351	0.477
Feasible Estimation														
Local Polynomial	0.38	0.27	0.123	0.156	0.077	0.109	0.056	0.093	0.114	0.142	0.114	0.142	0.114	0.142
B-splines	1	1	0.126	0.195	0.085	0.142	0.060	0.105	0.087	0.178	0.153	0.331	0.153	0.331
Partitioning	1	1	0.131	0.206	0.090	0.155	0.106	0.105	0.110	0.184	0.130	0.284	0.174	0.477
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.785	1.320	0.508	0.496	0.331	0.148	0.413	0.353	7.493	0.619	8.533	3.831
B-splines	216	27	1.225	1.175	0.571	0.539	0.504	0.161	0.612	0.458	2.295	2.005	4.466	4.340
Partitioning	216	27	1.272	1.207	0.475	0.543	0.958	1.318	0.552	2.008	2.090	7.569	3.754	8.329
Feasible Estimation														
Local Polynomial	0.29	0.26	1.345	1.327	0.469	0.489	0.063	0.096	0.431	0.325	0.431	0.325	3.805	3.800
B-splines	8	2	1.333	1.271	0.524	0.533	0.233	0.145	0.375	0.408	0.800	1.625	3.872	4.147
Partitioning	8	2	1.114	1.112	0.646	0.600	1.499	1.894	0.543	1.036	1.176	1.011	2.217	2.803
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.789	0.613	0.616	0.493	1.130	1.205	1.151	1.052	0.716	0.734	0.622	0.641
B-splines	27	27	0.850	0.635	0.669	0.510	0.925	1.344	1.002	1.209	0.893	0.784	0.541	0.798
Partitioning	27	27	0.741	1.264	0.578	0.998	0.800	1.352	0.780	2.299	0.691	7.462	0.764	8.473
Feasible Estimation														
Local Polynomial	0.28	0.28	0.750	0.593	0.587	0.477	1.216	1.225	1.262	1.173	0.580	0.669	0.598	0.514
B-splines	8	1	0.890	0.805	0.688	0.642	0.681	0.814	0.969	0.980	0.825	0.944	0.488	0.596
Partitioning	8	1	0.853	0.786	0.656	0.623	1.023	0.912	1.168	1.066	0.939	0.767	0.786	0.824

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.51: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.356	0.366	0.298	0.304	0.048	0.083	0.104	0.190	0.151	0.277	0.655	0.697
B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
Feasible Estimation														
Local Polynomial	0.43	0.29	0.497	0.504	0.404	0.409	0.053	0.087	0.164	0.226	0.662	0.679	1.291	1.300
B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.18	0.25	0.401	0.419	0.305	0.320	0.365	0.369	0.428	0.413	1.556	1.224	1.557	1.240
B-splines	27	8	0.398	0.416	0.308	0.317	0.368	0.369	0.358	0.452	0.645	1.348	0.685	1.329
Partitioning	27	8	0.489	0.571	0.375	0.435	0.245	2.043	0.432	2.504	0.832	3.185	1.519	3.821
Feasible Estimation														
Local Polynomial	0.36	0.26	0.369	0.380	0.278	0.288	0.381	0.385	0.345	0.369	0.309	0.338	0.328	0.353
B-splines	4	4	0.376	0.405	0.284	0.310	0.373	0.367	0.327	0.415	0.383	1.091	0.435	1.092
Partitioning	4	4	0.380	0.479	0.291	0.361	0.485	1.379	0.459	1.620	0.514	2.010	0.729	2.589
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.34	0.36	0.138	0.195	0.095	0.138	0.060	0.097	0.182	0.237	0.399	0.549	0.399	0.549
B-splines	1	1	0.106	0.186	0.074	0.131	0.046	0.087	0.091	0.240	0.197	0.754	0.197	0.754
Partitioning	1	1	0.107	0.203	0.077	0.149	0.046	0.088	0.091	0.221	0.134	0.426	0.169	0.745
Feasible Estimation														
Local Polynomial	0.37	0.27	0.100	0.137	0.066	0.095	0.051	0.081	0.157	0.203	0.157	0.203	0.157	0.203
B-splines	2	1	0.117	0.187	0.081	0.131	0.083	0.088	0.139	0.243	0.253	0.770	0.253	0.770
Partitioning	2	1	0.151	0.213	0.101	0.153	0.184	0.217	0.204	0.338	0.231	0.543	0.260	0.804
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.11	0.9	0.588	0.551	0.256	0.208	0.166	0.095	0.473	0.339	4.660	0.507	5.969	3.837
B-splines	64	27	0.534	0.478	0.262	0.265	0.229	0.120	0.506	0.482	2.002	4.443	4.233	5.714
Partitioning	64	27	0.741	0.816	0.498	0.443	0.909	0.481	1.916	6.145	2.477	19.806	4.284	3.994
Feasible Estimation														
Local Polynomial	0.32	0.26	0.561	0.544	0.184	0.202	0.053	0.082	0.361	0.415	0.361	0.415	3.818	3.821
B-splines	7	3	0.561	0.514	0.222	0.242	0.158	0.114	0.281	0.397	0.671	2.137	3.862	4.322
Partitioning	7	3	0.543	0.524	0.298	0.310	0.589	1.075	0.421	1.317	0.661	1.870	3.250	3.776
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.720	0.611	0.560	0.493	1.090	1.058	0.831	0.952	1.026	0.858	0.623	0.958
B-splines	27	27	0.770	0.640	0.598	0.512	0.995	1.173	0.717	0.975	1.229	4.150	0.802	3.842
Partitioning	27	27	0.679	1.103	0.519	0.868	0.757	0.535	0.618	6.429	0.994	19.892	1.636	2.082
Feasible Estimation														
Local Polynomial	0.28	0.27	0.713	0.616	0.558	0.495	1.199	1.194	0.984	1.178	0.882	0.697	0.362	0.534
B-splines	8	4	0.775	0.705	0.592	0.560	0.785	1.056	0.630	0.830	0.976	1.696	0.527	1.390
Partitioning	8	4	0.725	0.669	0.543	0.517	1.024	1.780	0.839	1.844	1.188	2.477	1.078	3.162

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.52: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.484	0.516	0.417	0.434	0.094	0.185	0.165	0.246	0.242	0.304	0.693	0.715
B-splines	1	1	0.488	0.573	0.419	0.470	0.093	0.235	0.157	0.340	0.281	0.486	0.709	0.809
Partitioning	1	1	0.179	0.399	0.141	0.307	0.090	0.206	0.151	0.345	0.199	0.475	0.242	0.671
Feasible Estimation														
Local Polynomial	0.38	0.28	0.662	0.686	0.540	0.558	0.122	0.213	0.213	0.270	0.676	0.695	1.299	1.309
B-splines	1	1	0.489	0.573	0.420	0.470	0.106	0.235	0.169	0.340	0.284	0.486	0.710	0.809
Partitioning	1	1	0.190	0.399	0.145	0.307	0.161	0.206	0.185	0.345	0.218	0.475	0.253	0.671
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.497	0.583	0.391	0.456	0.505	0.590	0.531	0.616	0.589	0.643	0.535	0.563
B-splines	8	8	0.447	0.565	0.350	0.441	0.612	0.621	0.566	0.619	0.544	0.602	0.427	0.540
Partitioning	8	8	0.594	1.136	0.466	0.874	1.040	8.320	1.028	5.138	0.816	3.056	0.633	1.467
Feasible Estimation														
Local Polynomial	0.34	0.26	0.398	0.438	0.309	0.343	0.373	0.405	0.342	0.378	0.357	0.388	0.327	0.364
B-splines	4	1	0.425	0.504	0.331	0.397	0.486	0.460	0.435	0.468	0.488	0.592	0.387	0.513
Partitioning	4	1	0.495	0.532	0.380	0.418	0.746	0.779	0.721	0.704	0.630	0.578	0.469	0.680
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.37	0.303	0.426	0.227	0.326	0.192	0.287	0.316	0.372	0.450	0.472	0.450	0.472
B-splines	1	1	0.208	0.370	0.154	0.281	0.097	0.232	0.151	0.331	0.271	0.473	0.271	0.473
Partitioning	1	1	0.209	0.403	0.158	0.310	0.097	0.207	0.151	0.346	0.206	0.477	0.261	0.682
Feasible Estimation														
Local Polynomial	0.35	0.26	0.199	0.273	0.143	0.207	0.124	0.208	0.204	0.259	0.204	0.259	0.204	0.259
B-splines	3	1	0.240	0.370	0.177	0.281	0.246	0.232	0.266	0.333	0.319	0.473	0.319	0.473
Partitioning	3	1	0.346	0.407	0.239	0.311	0.564	0.394	0.496	0.472	0.459	0.531	0.409	0.683
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.9	1.872	2.055	1.007	0.891	0.771	0.234	0.735	0.413	1.504	0.726	4.087	3.835
B-splines	125	27	2.039	1.959	1.032	0.999	0.361	0.362	0.560	0.598	1.620	1.427	4.138	4.047
Partitioning	125	27	1.510	1.547	0.944	0.892	0.660	0.048	0.941	1.063	1.786	3.566	5.967	7.999
Feasible Estimation														
Local Polynomial	0.3	0.26	2.083	2.070	0.846	0.882	0.135	0.211	0.506	0.408	0.506	0.408	3.795	3.793
B-splines	8	1	2.074	2.029	0.925	0.946	0.460	0.274	0.552	0.535	0.897	1.407	3.872	4.041
Partitioning	8	1	1.657	1.772	1.081	1.068	2.807	2.929	0.950	1.219	1.623	1.117	1.538	2.407
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.852	0.715	0.666	0.569	1.145	1.303	1.331	1.202	0.720	0.924	0.739	0.686
B-splines	27	27	0.930	0.773	0.741	0.613	0.859	1.452	1.192	1.374	0.895	0.826	0.644	0.760
Partitioning	27	27	1.109	1.669	0.873	1.331	1.340	0.385	2.324	1.463	0.987	4.319	0.922	7.798
Feasible Estimation														
Local Polynomial	0.28	0.26	0.783	0.605	0.614	0.486	1.200	1.214	1.364	1.184	0.514	0.709	0.706	0.547
B-splines	8	1	0.969	0.878	0.759	0.703	0.709	0.740	1.215	1.202	0.847	0.885	0.593	0.678
Partitioning	8	1	1.004	0.872	0.790	0.696	1.284	0.887	1.595	1.354	1.033	0.720	0.788	0.834

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.53: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.443	0.480	0.375	0.398	0.089	0.179	0.170	0.267	0.244	0.361	0.688	0.735
B-splines	1	1	0.449	0.542	0.379	0.437	0.088	0.212	0.160	0.367	0.305	0.682	0.708	0.899
Partitioning	1	1	0.174	0.399	0.136	0.301	0.086	0.204	0.158	0.361	0.207	0.567	0.249	0.874
Feasible Estimation														
Local Polynomial	0.38	0.27	0.606	0.633	0.494	0.514	0.112	0.192	0.239	0.297	0.687	0.710	1.306	1.319
B-splines	1	1	0.450	0.542	0.380	0.437	0.105	0.212	0.170	0.367	0.310	0.682	0.710	0.899
Partitioning	1	1	0.194	0.399	0.144	0.301	0.210	0.204	0.222	0.361	0.245	0.567	0.278	0.874
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.501	0.582	0.389	0.450	0.466	0.533	0.502	0.542	0.705	0.719	0.724	0.729
B-splines	8	8	0.450	0.571	0.350	0.441	0.568	0.561	0.511	0.586	0.566	0.813	0.537	0.839
Partitioning	8	8	0.596	1.135	0.467	0.873	0.957	6.199	0.955	4.840	0.855	3.580	0.858	2.477
Feasible Estimation														
Local Polynomial	0.34	0.25	0.399	0.439	0.309	0.343	0.387	0.416	0.373	0.410	0.363	0.396	0.358	0.396
B-splines	5	1	0.432	0.510	0.335	0.398	0.495	0.456	0.436	0.470	0.511	0.724	0.487	0.732
Partitioning	5	1	0.519	0.571	0.398	0.434	0.755	1.403	0.714	0.889	0.684	1.062	0.679	0.982
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.34	0.38	0.277	0.395	0.201	0.296	0.146	0.232	0.294	0.346	0.532	0.555	0.532	0.555
B-splines	1	1	0.195	0.365	0.143	0.270	0.090	0.209	0.159	0.355	0.298	0.662	0.298	0.662
Partitioning	1	1	0.196	0.402	0.148	0.304	0.090	0.205	0.158	0.363	0.213	0.567	0.269	0.899
Feasible Estimation														
Local Polynomial	0.34	0.25	0.185	0.264	0.133	0.197	0.116	0.192	0.234	0.285	0.234	0.285	0.234	0.285
B-splines	4	1	0.245	0.367	0.179	0.271	0.262	0.215	0.318	0.362	0.408	0.665	0.408	0.665
Partitioning	4	1	0.390	0.428	0.274	0.314	0.622	0.714	0.550	0.659	0.546	0.766	0.558	0.957
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.1	0.9	1.209	1.337	0.676	0.550	0.517	0.215	0.653	0.420	2.118	0.693	4.419	3.844
B-splines	125	27	1.338	1.264	0.707	0.676	0.291	0.305	0.554	0.601	2.127	2.164	4.424	4.442
Partitioning	125	27	1.528	1.569	0.914	0.830	1.193	2.635	1.729	4.020	2.222	15.117	4.131	15.210
Feasible Estimation														
Local Polynomial	0.3	0.25	1.353	1.342	0.500	0.539	0.123	0.193	0.475	0.413	0.475	0.413	3.809	3.807
B-splines	8	3	1.353	1.310	0.576	0.616	0.392	0.270	0.517	0.528	0.894	1.758	3.899	4.221
Partitioning	8	3	1.198	1.219	0.745	0.735	1.650	3.276	0.857	2.432	1.334	1.855	2.323	3.086
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.822	0.723	0.641	0.575	1.139	1.228	1.171	1.096	0.843	0.918	0.783	0.872
B-splines	27	27	0.905	0.787	0.712	0.622	0.942	1.371	1.029	1.263	1.022	1.085	0.759	1.126
Partitioning	27	27	1.091	1.613	0.857	1.285	0.930	2.643	1.041	4.209	1.168	14.812	1.331	15.577
Feasible Estimation														
Local Polynomial	0.29	0.26	0.774	0.624	0.605	0.501	1.212	1.220	1.265	1.205	0.629	0.708	0.619	0.575
B-splines	8	1	0.922	0.860	0.711	0.682	0.786	0.864	0.998	1.023	0.933	1.081	0.641	0.839
Partitioning	8	1	0.960	0.869	0.744	0.684	1.264	1.371	1.320	1.277	1.150	1.103	0.997	1.210

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.54: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.383	0.423	0.316	0.344	0.091	0.159	0.197	0.363	0.290	0.529	0.699	0.828
B-splines	1	1	0.391	0.496	0.321	0.388	0.090	0.177	0.185	0.489	0.396	1.550	0.752	1.685
Partitioning	1	1	0.180	0.401	0.139	0.294	0.089	0.175	0.180	0.438	0.253	0.851	0.303	1.409
Feasible Estimation														
Local Polynomial	0.37	0.27	0.516	0.547	0.418	0.440	0.103	0.166	0.327	0.422	0.719	0.768	1.322	1.349
B-splines	2	1	0.401	0.497	0.328	0.389	0.155	0.179	0.272	0.497	0.496	1.579	0.811	1.719
Partitioning	2	1	0.274	0.420	0.185	0.302	0.345	0.428	0.397	0.679	0.424	1.071	0.482	1.534
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.492	0.594	0.371	0.441	0.391	0.416	0.594	0.635	1.385	1.672	1.387	1.681
B-splines	8	8	0.447	0.568	0.341	0.426	0.441	0.442	0.524	0.758	0.734	2.517	0.790	2.492
Partitioning	8	8	0.585	1.133	0.456	0.863	0.876	3.964	0.901	4.797	1.009	6.217	1.308	7.213
Feasible Estimation														
Local Polynomial	0.33	0.25	0.396	0.437	0.302	0.334	0.392	0.409	0.450	0.519	0.423	0.500	0.437	0.508
B-splines	7	6	0.441	0.552	0.336	0.415	0.427	0.431	0.505	0.714	0.706	2.279	0.756	2.265
Partitioning	7	6	0.560	1.008	0.432	0.740	0.817	3.397	0.832	3.998	0.934	5.375	1.215	6.184
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.41	0.41	0.231	0.346	0.165	0.248	0.105	0.177	0.305	0.442	0.625	0.956	0.625	0.956
B-splines	1	1	0.188	0.361	0.138	0.256	0.089	0.174	0.181	0.480	0.392	1.509	0.392	1.509
Partitioning	1	1	0.189	0.402	0.144	0.295	0.089	0.175	0.181	0.439	0.257	0.851	0.313	1.431
Feasible Estimation														
Local Polynomial	0.33	0.25	0.173	0.255	0.123	0.182	0.105	0.166	0.329	0.417	0.329	0.417	0.329	0.417
B-splines	6	5	0.260	0.413	0.190	0.291	0.261	0.247	0.425	0.625	0.648	2.135	0.648	2.135
Partitioning	6	5	0.459	0.880	0.339	0.604	0.662	2.917	0.696	3.451	0.760	4.610	0.815	5.435
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.13	0.9	0.632	0.591	0.387	0.277	0.267	0.166	0.746	0.453	4.138	0.672	5.652	3.864
B-splines	64	27	0.658	0.666	0.396	0.427	0.456	0.236	0.872	0.779	2.471	6.213	4.448	7.154
Partitioning	64	27	1.297	1.321	0.942	0.776	1.819	0.961	3.752	12.288	4.721	39.529	5.277	4.588
Feasible Estimation														
Local Polynomial	0.31	0.25	0.579	0.582	0.224	0.267	0.106	0.167	0.459	0.544	0.459	0.544	3.828	3.835
B-splines	8	6	0.606	0.627	0.294	0.370	0.296	0.263	0.481	0.723	0.890	3.024	3.908	4.749
Partitioning	8	6	0.691	1.019	0.459	0.726	0.865	3.380	0.786	4.128	0.980	5.223	3.319	6.607
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.755	0.705	0.585	0.560	1.095	1.071	0.892	1.043	1.250	1.368	0.944	1.438
B-splines	27	27	0.829	0.790	0.643	0.615	1.002	1.189	0.799	1.152	1.572	6.088	1.259	5.874
Partitioning	27	27	1.051	1.515	0.809	1.200	0.802	0.992	0.950	12.471	1.687	39.465	2.983	3.067
Feasible Estimation														
Local Polynomial	0.29	0.25	0.733	0.646	0.572	0.517	1.197	1.186	1.011	1.236	0.944	0.782	0.455	0.648
B-splines	8	6	0.809	0.778	0.617	0.612	0.832	1.152	0.741	1.074	1.152	2.638	0.799	2.463
Partitioning	8	6	0.846	1.074	0.643	0.814	1.208	3.553	1.056	4.091	1.382	5.518	1.323	6.547

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.55: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.458	0.462	0.403	0.404	0.034	0.073	0.062	0.092	0.091	0.116	0.648	0.656
B-splines	1	1	0.459	0.469	0.404	0.407	0.034	0.093	0.060	0.124	0.103	0.177	0.650	0.667
Partitioning	1	1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
Feasible Estimation														
Local Polynomial	0.46	0.31	0.645	0.648	0.526	0.528	0.043	0.081	0.080	0.106	0.648	0.653	1.285	1.288
B-splines	1	1	0.459	0.469	0.404	0.407	0.034	0.093	0.060	0.124	0.103	0.177	0.650	0.667
Partitioning	1	1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.382	0.396	0.297	0.307	0.468	0.497	0.392	0.432	0.370	0.402	0.287	0.319
B-splines	27	8	0.380	0.386	0.296	0.297	0.403	0.495	0.335	0.406	0.384	0.377	0.257	0.268
Partitioning	27	8	0.374	0.421	0.292	0.322	0.321	2.568	0.327	1.655	0.294	0.903	0.301	0.417
Feasible Estimation														
Local Polynomial	0.38	0.28	0.369	0.374	0.281	0.287	0.357	0.353	0.284	0.293	0.298	0.302	0.265	0.274
B-splines	1	1	0.370	0.380	0.283	0.294	0.379	0.408	0.271	0.337	0.340	0.381	0.246	0.261
Partitioning	1	1	0.370	0.375	0.284	0.291	0.388	0.394	0.312	0.534	0.335	0.280	0.262	0.249
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.22	0.3	0.156	0.202	0.112	0.151	0.095	0.127	0.131	0.170	0.166	0.216	0.166	0.216
B-splines	8	8	0.138	0.186	0.096	0.138	0.151	0.153	0.142	0.159	0.135	0.166	0.135	0.166
Partitioning	8	8	0.187	0.400	0.145	0.305	0.356	2.133	0.278	1.308	0.250	0.832	0.198	0.410
Feasible Estimation														
Local Polynomial	0.39	0.28	0.130	0.146	0.074	0.094	0.046	0.072	0.072	0.091	0.072	0.091	0.072	0.091
B-splines	1	1	0.125	0.162	0.080	0.116	0.049	0.085	0.057	0.114	0.096	0.161	0.096	0.161
Partitioning	1	1	0.126	0.153	0.081	0.117	0.051	0.083	0.060	0.122	0.100	0.171	0.142	0.255
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.07	0.9	1.844	2.068	0.813	0.823	0.361	0.133	0.275	0.237	0.945	0.472	3.936	3.837
B-splines	343	64	2.009	1.964	0.876	0.859	0.179	0.225	0.268	0.317	1.137	0.825	4.003	3.904
Partitioning	343	64	1.162	1.130	0.543	0.505	0.115	0.000	0.466	1.182	8.935	8.965	2.670	3.624
Feasible Estimation														
Local Polynomial	0.29	0.26	2.084	2.076	0.803	0.816	0.049	0.077	0.317	0.226	0.317	0.226	3.817	3.800
B-splines	8	2	2.076	2.047	0.846	0.848	0.202	0.119	0.307	0.310	0.606	0.970	3.865	3.951
Partitioning	8	2	1.618	1.740	1.022	0.984	2.474	1.815	0.450	0.501	1.492	0.562	1.313	2.095
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.818	0.582	0.639	0.465	1.141	1.285	1.306	1.119	0.578	0.747	0.619	0.468
B-splines	27	27	0.876	0.595	0.698	0.476	0.843	1.435	1.159	1.296	0.749	0.628	0.468	0.573
Partitioning	27	27	0.726	0.748	0.575	0.581	0.811	2.888	0.871	2.153	0.341	1.036	0.396	0.650
Feasible Estimation														
Local Polynomial	0.25	0.28	0.725	0.567	0.570	0.455	1.212	1.213	1.385	1.156	0.438	0.667	0.712	0.487
B-splines	8	1	0.936	0.820	0.735	0.661	0.538	0.711	1.156	1.166	0.716	0.724	0.473	0.501
Partitioning	8	1	0.898	0.797	0.704	0.639	0.922	0.862	1.380	1.321	0.747	0.534	0.590	0.532

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.56: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.419	0.423	0.362	0.364	0.036	0.065	0.067	0.104	0.095	0.135	0.646	0.649
B-splines	1	1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
Feasible Estimation														
Local Polynomial	0.45	0.31	0.590	0.592	0.481	0.484	0.042	0.073	0.094	0.119	0.648	0.653	1.285	1.287
B-splines	1	1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.390	0.403	0.303	0.312	0.424	0.442	0.368	0.391	0.386	0.412	0.366	0.386
B-splines	27	8	0.386	0.394	0.300	0.305	0.389	0.438	0.329	0.379	0.374	0.383	0.330	0.357
Partitioning	27	8	0.373	0.419	0.292	0.320	0.266	1.835	0.292	1.465	0.342	1.006	0.396	0.711
Feasible Estimation														
Local Polynomial	0.37	0.27	0.372	0.377	0.283	0.288	0.370	0.373	0.301	0.309	0.289	0.294	0.283	0.291
B-splines	2	1	0.373	0.385	0.285	0.298	0.380	0.393	0.304	0.333	0.314	0.381	0.287	0.342
Partitioning	2	1	0.373	0.383	0.285	0.296	0.401	0.402	0.357	0.469	0.320	0.302	0.325	0.346
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.25	0.32	0.139	0.183	0.097	0.133	0.069	0.098	0.130	0.155	0.208	0.236	0.208	0.236
B-splines	8	1	0.126	0.151	0.088	0.107	0.124	0.071	0.144	0.126	0.165	0.229	0.165	0.229
Partitioning	8	1	0.186	0.150	0.145	0.113	0.318	0.072	0.275	0.131	0.240	0.199	0.225	0.335
Feasible Estimation														
Local Polynomial	0.38	0.28	0.112	0.130	0.065	0.085	0.043	0.067	0.085	0.103	0.085	0.103	0.085	0.103
B-splines	1	1	0.111	0.151	0.072	0.107	0.046	0.071	0.069	0.126	0.113	0.229	0.113	0.229
Partitioning	1	1	0.115	0.150	0.075	0.113	0.092	0.072	0.075	0.131	0.105	0.199	0.142	0.335
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.284	1.342	0.491	0.472	0.235	0.096	0.265	0.254	1.381	0.447	4.003	3.809
B-splines	216	27	1.292	1.263	0.529	0.511	0.345	0.114	0.409	0.323	1.545	1.350	4.065	3.990
Partitioning	216	27	1.034	0.749	0.579	0.584	2.686	0.602	2.256	1.060	1.771	2.178	3.608	2.427
Feasible Estimation														
Local Polynomial	0.29	0.26	1.355	1.346	0.454	0.468	0.046	0.069	0.316	0.237	0.316	0.237	3.802	3.790
B-splines	8	3	1.349	1.313	0.493	0.501	0.161	0.104	0.268	0.310	0.586	1.199	3.820	3.942
Partitioning	8	3	1.127	1.083	0.640	0.558	1.390	1.710	0.437	0.694	1.082	0.744	2.128	2.509
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.789	0.601	0.616	0.485	1.125	1.200	1.147	1.047	0.663	0.680	0.570	0.567
B-splines	27	27	0.849	0.615	0.669	0.496	0.917	1.340	0.994	1.206	0.837	0.616	0.446	0.659
Partitioning	27	27	0.701	0.748	0.545	0.583	0.780	0.661	0.720	1.067	0.444	2.060	0.469	2.016
Feasible Estimation														
Local Polynomial	0.25	0.28	0.720	0.589	0.566	0.474	1.225	1.225	1.286	1.166	0.540	0.662	0.616	0.499
B-splines	8	1	0.890	0.802	0.688	0.641	0.648	0.808	0.961	0.968	0.791	0.854	0.431	0.475
Partitioning	8	1	0.848	0.780	0.651	0.619	0.947	0.907	1.150	1.062	0.877	0.729	0.704	0.665

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.57: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2,2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.353	0.358	0.296	0.299	0.033	0.055	0.071	0.134	0.101	0.188	0.648	0.669
B-splines	1	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
Feasible Estimation														
Local Polynomial	0.43	0.3	0.495	0.499	0.403	0.406	0.037	0.060	0.113	0.155	0.655	0.669	1.290	1.301
B-splines	1	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.384	0.393	0.292	0.299	0.352	0.351	0.363	0.350	0.722	0.553	0.743	0.590
B-splines	27	8	0.379	0.389	0.291	0.296	0.362	0.350	0.319	0.364	0.462	0.714	0.529	0.744
Partitioning	27	8	0.369	0.413	0.285	0.310	0.218	1.289	0.304	1.434	0.524	1.720	0.775	2.121
Feasible Estimation														
Local Polynomial	0.36	0.27	0.366	0.371	0.274	0.279	0.376	0.378	0.320	0.328	0.293	0.310	0.302	0.311
B-splines	4	4	0.369	0.384	0.277	0.292	0.360	0.353	0.301	0.352	0.320	0.644	0.374	0.681
Partitioning	4	4	0.360	0.397	0.274	0.300	0.453	0.975	0.439	1.061	0.469	1.297	0.709	1.743
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.31	0.34	0.110	0.150	0.074	0.105	0.046	0.074	0.143	0.175	0.319	0.350	0.319	0.350
B-splines	8	1	0.107	0.136	0.076	0.094	0.109	0.061	0.163	0.173	0.240	0.470	0.240	0.470
Partitioning	8	1	0.182	0.145	0.141	0.105	0.271	0.062	0.273	0.154	0.293	0.281	0.305	0.525
Feasible Estimation														
Local Polynomial	0.37	0.27	0.083	0.106	0.051	0.070	0.037	0.057	0.115	0.142	0.115	0.142	0.115	0.142
B-splines	2	1	0.091	0.136	0.061	0.094	0.056	0.061	0.093	0.173	0.165	0.470	0.165	0.470
Partitioning	2	1	0.112	0.145	0.074	0.105	0.134	0.062	0.136	0.154	0.155	0.281	0.183	0.525
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.1	0.9	0.485	0.554	0.222	0.183	0.137	0.065	0.331	0.248	2.492	0.368	4.498	3.820
B-splines	125	27	0.528	0.491	0.237	0.226	0.081	0.089	0.284	0.344	1.975	2.346	4.276	4.467
Partitioning	125	27	0.741	0.748	0.491	0.536	0.260	0.269	0.866	0.791	1.790	6.145	3.850	397.445
Feasible Estimation														
Local Polynomial	0.31	0.26	0.560	0.551	0.168	0.182	0.038	0.058	0.265	0.297	0.265	0.297	3.799	3.798
B-splines	8	3	0.560	0.525	0.195	0.212	0.121	0.084	0.219	0.300	0.498	1.636	3.833	4.146
Partitioning	8	3	0.525	0.483	0.263	0.256	0.467	0.891	0.326	0.726	0.570	1.086	3.154	3.149
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.719	0.602	0.559	0.487	1.087	1.049	0.812	0.924	0.896	0.657	0.498	0.752
B-splines	27	27	0.767	0.622	0.596	0.501	0.990	1.169	0.692	0.960	1.020	1.510	0.566	1.330
Partitioning	27	27	0.629	0.766	0.474	0.584	0.756	0.357	0.485	0.841	0.715	6.931	0.885	306.734
Feasible Estimation														
Local Polynomial	0.26	0.27	0.696	0.612	0.548	0.492	1.207	1.192	1.014	1.157	0.823	0.685	0.361	0.498
B-splines	8	7	0.773	0.664	0.591	0.533	0.757	1.160	0.596	0.936	0.913	1.027	0.414	0.926
Partitioning	8	7	0.716	0.512	0.532	0.392	0.948	1.441	0.779	1.398	1.124	1.633	0.993	1.999

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.58: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.469	0.487	0.409	0.418	0.063	0.136	0.117	0.172	0.171	0.218	0.665	0.685
B-splines	1	1	0.471	0.518	0.410	0.436	0.063	0.174	0.111	0.232	0.195	0.330	0.669	0.725
Partitioning	1	1	0.125	0.283	0.098	0.217	0.060	0.154	0.108	0.239	0.143	0.337	0.170	0.451
Feasible Estimation														
Local Polynomial	0.39	0.28	0.652	0.665	0.532	0.542	0.084	0.153	0.148	0.192	0.661	0.672	1.293	1.299
B-splines	1	1	0.472	0.518	0.411	0.436	0.073	0.174	0.117	0.232	0.196	0.330	0.669	0.725
Partitioning	1	1	0.132	0.283	0.101	0.217	0.120	0.154	0.122	0.239	0.153	0.337	0.173	0.451
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.443	0.493	0.348	0.386	0.491	0.548	0.447	0.512	0.477	0.509	0.409	0.444
B-splines	27	8	0.439	0.474	0.349	0.370	0.427	0.562	0.381	0.486	0.475	0.476	0.376	0.390
Partitioning	27	8	0.682	0.811	0.529	0.619	0.538	4.500	0.598	2.836	0.580	1.708	0.591	0.822
Feasible Estimation														
Local Polynomial	0.35	0.26	0.381	0.403	0.294	0.314	0.362	0.378	0.309	0.334	0.325	0.342	0.292	0.318
B-splines	4	1	0.395	0.438	0.306	0.345	0.453	0.437	0.380	0.387	0.416	0.474	0.312	0.380
Partitioning	4	1	0.429	0.449	0.332	0.353	0.607	0.419	0.581	0.605	0.487	0.411	0.377	0.463
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.27	0.34	0.235	0.325	0.175	0.248	0.152	0.219	0.229	0.283	0.309	0.355	0.309	0.355
B-splines	8	1	0.215	0.272	0.162	0.205	0.301	0.170	0.284	0.228	0.270	0.323	0.270	0.323
Partitioning	8	1	0.362	0.289	0.283	0.222	0.645	0.157	0.556	0.240	0.500	0.338	0.397	0.465
Feasible Estimation														
Local Polynomial	0.35	0.26	0.163	0.210	0.110	0.155	0.088	0.152	0.146	0.186	0.146	0.186	0.146	0.186
B-splines	3	1	0.184	0.272	0.133	0.205	0.191	0.170	0.195	0.228	0.224	0.323	0.224	0.323
Partitioning	3	1	0.250	0.289	0.174	0.222	0.365	0.157	0.328	0.240	0.311	0.338	0.286	0.465
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.957	2.074	0.956	0.852	0.552	0.175	0.457	0.278	1.047	0.507	3.965	3.837
B-splines	216	27	2.060	2.024	0.979	0.935	0.831	0.265	0.609	0.406	1.194	0.976	4.013	3.950
Partitioning	216	27	1.444	1.457	0.961	1.120	2.111	5.544	2.865	3.849	3.373	2.078	2.663	1.680
Feasible Estimation														
Local Polynomial	0.3	0.25	2.087	2.081	0.819	0.844	0.095	0.152	0.340	0.281	0.340	0.281	3.816	3.805
B-splines	8	2	2.083	2.059	0.873	0.892	0.331	0.200	0.397	0.376	0.653	1.006	3.874	3.958
Partitioning	8	2	1.647	1.767	1.053	1.023	2.524	2.178	0.661	0.899	1.561	0.788	1.348	2.148
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.835	0.645	0.652	0.515	1.147	1.302	1.314	1.145	0.636	0.812	0.668	0.565
B-splines	27	27	0.903	0.680	0.720	0.543	0.855	1.453	1.171	1.316	0.800	0.695	0.541	0.644
Partitioning	27	27	0.924	1.452	0.732	1.126	0.920	5.644	1.010	4.058	0.602	2.047	0.646	1.299
Feasible Estimation														
Local Polynomial	0.26	0.26	0.746	0.581	0.585	0.467	1.211	1.204	1.374	1.169	0.471	0.684	0.708	0.514
B-splines	8	1	0.951	0.849	0.746	0.681	0.602	0.732	1.178	1.179	0.752	0.777	0.528	0.571
Partitioning	8	1	0.950	0.834	0.747	0.668	1.077	0.877	1.459	1.346	0.865	0.612	0.678	0.657

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.59: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.432	0.451	0.369	0.380	0.067	0.122	0.126	0.195	0.180	0.254	0.663	0.682
B-splines	1	1	0.435	0.484	0.371	0.401	0.067	0.144	0.120	0.257	0.221	0.465	0.680	0.781
Partitioning	1	1	0.129	0.284	0.101	0.215	0.066	0.137	0.118	0.254	0.155	0.394	0.180	0.575
Feasible Estimation														
Local Polynomial	0.38	0.28	0.598	0.612	0.488	0.498	0.083	0.139	0.178	0.218	0.669	0.682	1.297	1.304
B-splines	1	1	0.436	0.484	0.372	0.401	0.078	0.144	0.133	0.257	0.226	0.465	0.681	0.781
Partitioning	1	1	0.144	0.284	0.107	0.215	0.158	0.137	0.150	0.254	0.185	0.394	0.190	0.575
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.450	0.496	0.351	0.386	0.438	0.480	0.440	0.474	0.553	0.558	0.541	0.544
B-splines	27	8	0.444	0.480	0.352	0.373	0.401	0.485	0.382	0.484	0.523	0.567	0.497	0.555
Partitioning	27	8	0.680	0.809	0.531	0.617	0.410	3.201	0.535	2.547	0.672	1.865	0.784	1.322
Feasible Estimation														
Local Polynomial	0.34	0.26	0.385	0.407	0.297	0.316	0.377	0.393	0.335	0.354	0.329	0.348	0.318	0.338
B-splines	5	1	0.403	0.444	0.311	0.348	0.437	0.411	0.394	0.398	0.424	0.548	0.383	0.525
Partitioning	5	1	0.442	0.461	0.343	0.360	0.597	0.522	0.629	0.583	0.511	0.507	0.532	0.619
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.36	0.218	0.301	0.158	0.224	0.113	0.171	0.233	0.267	0.401	0.405	0.401	0.405
B-splines	1	1	0.157	0.266	0.114	0.197	0.071	0.141	0.119	0.252	0.216	0.459	0.216	0.459
Partitioning	1	1	0.157	0.288	0.116	0.218	0.071	0.139	0.119	0.257	0.166	0.395	0.209	0.604
Feasible Estimation														
Local Polynomial	0.34	0.26	0.151	0.201	0.105	0.147	0.085	0.138	0.174	0.208	0.174	0.208	0.174	0.208
B-splines	4	1	0.187	0.266	0.136	0.197	0.185	0.141	0.230	0.253	0.283	0.459	0.283	0.459
Partitioning	4	1	0.280	0.292	0.198	0.219	0.435	0.197	0.405	0.264	0.358	0.417	0.349	0.607
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.9	1.255	1.351	0.615	0.508	0.379	0.139	0.451	0.302	1.481	0.496	4.046	3.817
B-splines	125	27	1.349	1.305	0.618	0.601	0.206	0.217	0.400	0.431	1.498	1.431	4.057	4.021
Partitioning	125	27	1.434	1.476	1.084	1.149	1.216	1.203	1.570	2.108	1.813	4.319	3.341	4.205
Feasible Estimation														
Local Polynomial	0.3	0.26	1.359	1.354	0.475	0.501	0.090	0.139	0.348	0.300	0.348	0.300	3.801	3.790
B-splines	8	3	1.359	1.334	0.527	0.555	0.271	0.182	0.369	0.391	0.655	1.257	3.832	3.968
Partitioning	8	3	1.169	1.171	0.692	0.648	1.480	2.050	0.652	1.153	1.161	1.099	2.173	2.602
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.806	0.659	0.629	0.530	1.126	1.208	1.159	1.073	0.733	0.777	0.657	0.686
B-splines	27	27	0.877	0.698	0.691	0.558	0.920	1.348	1.010	1.234	0.908	0.748	0.576	0.788
Partitioning	27	27	0.903	1.475	0.710	1.149	0.836	1.242	0.843	2.094	0.729	4.199	0.823	3.981
Feasible Estimation														
Local Polynomial	0.26	0.26	0.740	0.602	0.581	0.484	1.223	1.214	1.274	1.180	0.582	0.684	0.614	0.530
B-splines	8	1	0.906	0.828	0.700	0.659	0.678	0.846	0.991	0.997	0.840	0.927	0.515	0.620
Partitioning	8	1	0.903	0.822	0.697	0.650	1.064	1.103	1.241	1.154	0.964	0.894	0.810	0.866

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.60: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2, 2)$, Uniform Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		Point Estimation RMSE (0.1,0.5,0.5)		Point Estimation RMSE (0.1,0.1,0.5)		Point Estimation RMSE (0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.366	0.387	0.305	0.319	0.063	0.106	0.137	0.262	0.194	0.360	0.670	0.736
B-splines	1	1	0.370	0.428	0.307	0.344	0.063	0.123	0.132	0.344	0.276	0.946	0.705	1.153
Partitioning	1	1	0.124	0.281	0.096	0.204	0.062	0.121	0.131	0.302	0.170	0.561	0.211	0.935
Feasible Estimation														
Local Polynomial	0.37	0.27	0.505	0.521	0.410	0.422	0.073	0.117	0.232	0.290	0.692	0.721	1.313	1.332
B-splines	2	1	0.374	0.428	0.310	0.344	0.105	0.123	0.180	0.344	0.316	0.946	0.722	1.153
Partitioning	2	1	0.179	0.281	0.122	0.204	0.209	0.121	0.252	0.302	0.274	0.561	0.301	0.935
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.441	0.479	0.334	0.363	0.368	0.380	0.486	0.476	0.972	0.850	0.985	0.870
B-splines	27	8	0.438	0.475	0.340	0.359	0.369	0.392	0.402	0.550	0.805	1.317	0.845	1.335
Partitioning	27	8	0.678	0.807	0.519	0.605	0.287	2.374	0.567	2.651	0.996	3.173	1.513	3.729
Feasible Estimation														
Local Polynomial	0.33	0.25	0.379	0.400	0.287	0.305	0.380	0.388	0.379	0.405	0.365	0.405	0.364	0.392
B-splines	7	6	0.401	0.463	0.305	0.350	0.388	0.380	0.393	0.516	0.510	1.219	0.569	1.241
Partitioning	7	6	0.453	0.705	0.350	0.515	0.658	1.941	0.657	2.195	0.731	2.550	1.014	3.195
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.38	0.39	0.178	0.257	0.124	0.183	0.078	0.133	0.243	0.326	0.504	0.629	0.504	0.629
B-splines	1	1	0.137	0.257	0.100	0.179	0.064	0.122	0.131	0.345	0.273	0.941	0.273	0.941
Partitioning	1	1	0.137	0.283	0.103	0.205	0.064	0.122	0.131	0.304	0.175	0.561	0.222	0.968
Feasible Estimation														
Local Polynomial	0.33	0.25	0.131	0.185	0.091	0.131	0.074	0.117	0.240	0.289	0.240	0.289	0.240	0.289
B-splines	6	3	0.183	0.274	0.133	0.190	0.192	0.154	0.288	0.374	0.439	1.047	0.439	1.047
Partitioning	6	3	0.318	0.469	0.233	0.303	0.461	1.202	0.482	1.312	0.518	1.507	0.548	2.059
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.12	0.9	0.565	0.574	0.318	0.228	0.224	0.112	0.555	0.334	2.406	0.476	4.487	3.832
B-splines	64	27	0.605	0.590	0.314	0.331	0.354	0.175	0.617	0.530	1.771	2.904	4.200	4.793
Partitioning	64	27	1.033	1.412	0.770	1.050	1.222	0.539	1.662	1.579	3.031	12.237	7.126	840.145
Feasible Estimation														
Local Polynomial	0.31	0.25	0.569	0.569	0.194	0.223	0.075	0.117	0.335	0.382	0.335	0.382	3.799	3.799
B-splines	8	5	0.582	0.580	0.241	0.289	0.223	0.186	0.359	0.497	0.647	1.972	3.853	4.277
Partitioning	8	5	0.607	0.728	0.365	0.482	0.634	1.866	0.578	2.031	0.760	2.396	3.209	3.842
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.736	0.651	0.572	0.523	1.086	1.052	0.839	0.965	1.023	0.901	0.701	0.971
B-splines	27	27	0.798	0.703	0.619	0.557	0.991	1.174	0.731	1.033	1.211	2.233	0.864	2.122
Partitioning	27	27	0.848	1.421	0.651	1.092	0.774	0.587	0.681	1.606	1.105	12.939	1.568	747.807
Feasible Estimation														
Local Polynomial	0.27	0.25	0.712	0.624	0.559	0.501	1.201	1.179	1.002	1.176	0.881	0.736	0.390	0.550
B-splines	8	7	0.790	0.720	0.603	0.572	0.772	1.155	0.658	1.006	1.005	1.500	0.583	1.431
Partitioning	8	7	0.779	0.830	0.586	0.631	1.046	2.306	0.910	2.476	1.238	2.838	1.138	3.512

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

C.3.2 Quantile Cell Boundaries

Table C.61: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.462	0.468	0.405	0.406	0.051	0.099	0.090	0.133	0.131	0.164	0.658	0.663
B-splines	1	1	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
Feasible Estimation														
Local Polynomial	0.45	0.31	0.648	0.653	0.529	0.533	0.062	0.115	0.118	0.155	0.651	0.659	1.285	1.289
B-splines	1	1	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.401	0.426	0.314	0.333	0.470	0.508	0.426	0.482	0.416	0.475	0.344	0.371
B-splines	27	8	0.399	0.413	0.314	0.321	0.398	0.507	0.352	0.455	0.426	0.422	0.316	0.328
Partitioning	27	8	0.495	0.576	0.387	0.444	0.391	3.315	0.427	2.301	0.453	1.313	0.501	0.760
Feasible Estimation														
Local Polynomial	0.38	0.28	0.373	0.383	0.286	0.296	0.360	0.361	0.293	0.308	0.308	0.315	0.275	0.290
B-splines	1	1	0.376	0.398	0.289	0.312	0.375	0.413	0.270	0.366	0.359	0.425	0.262	0.311
Partitioning	1	1	0.377	0.398	0.290	0.312	0.381	0.400	0.299	0.553	0.346	0.335	0.269	0.354
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.25	0.32	0.188	0.253	0.138	0.191	0.120	0.166	0.178	0.221	0.231	0.287	0.231	0.287
B-splines	8	1	0.167	0.204	0.122	0.152	0.186	0.116	0.191	0.166	0.193	0.236	0.193	0.236
Partitioning	8	1	0.258	0.207	0.201	0.159	0.432	0.106	0.384	0.175	0.340	0.241	0.284	0.357
Feasible Estimation														
Local Polynomial	0.39	0.28	0.142	0.169	0.087	0.117	0.061	0.100	0.101	0.125	0.101	0.125	0.101	0.125
B-splines	1	1	0.140	0.204	0.094	0.152	0.058	0.116	0.077	0.166	0.137	0.236	0.137	0.236
Partitioning	1	1	0.142	0.207	0.097	0.159	0.069	0.106	0.079	0.175	0.117	0.241	0.163	0.357
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.07	0.9	1.791	2.044	0.814	0.847	0.644	0.180	0.513	0.357	1.378	0.678	4.044	3.830
B-splines	216	27	1.902	1.890	0.873	0.873	0.394	0.183	0.493	0.448	1.565	1.326	4.118	4.010
Partitioning	216	27	1.926	1.762	0.757	0.758	1.572	1.693	0.673	2.185	0.784	2.899	3.748	4.395
Feasible Estimation														
Local Polynomial	0.29	0.26	2.077	2.061	0.820	0.839	0.070	0.107	0.475	0.336	0.475	0.336	3.792	3.787
B-splines	8	1	2.061	2.005	0.882	0.880	0.244	0.156	0.390	0.445	0.838	1.343	3.864	4.024
Partitioning	8	1	1.599	1.743	1.015	1.018	2.269	1.146	0.594	0.490	1.548	0.751	1.458	2.308
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.819	0.598	0.639	0.477	1.138	1.281	1.305	1.141	0.617	0.792	0.642	0.509
B-splines	27	27	0.845	0.612	0.676	0.489	0.875	1.383	1.185	1.282	0.761	0.712	0.548	0.580
Partitioning	27	27	0.709	1.395	0.555	1.093	0.946	1.727	1.040	2.383	0.518	3.710	0.599	3.436
Feasible Estimation														
Local Polynomial	0.27	0.28	0.756	0.573	0.592	0.460	1.205	1.219	1.359	1.155	0.476	0.681	0.690	0.493
B-splines	8	1	0.936	0.822	0.734	0.661	0.622	0.716	1.175	1.166	0.756	0.782	0.497	0.545
Partitioning	8	1	0.900	0.801	0.705	0.641	0.953	0.870	1.318	1.322	0.795	0.588	0.628	0.604

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.62: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 3.1									
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.421	0.429	0.362	0.366	0.047	0.094	0.090	0.142	0.130	0.191	0.654	0.667
B-splines	1	1	0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
Partitioning	1	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
Feasible Estimation														
Local Polynomial	0.44	0.3	0.591	0.596	0.482	0.486	0.057	0.100	0.126	0.164	0.655	0.663	1.289	1.292
B-splines	1	1	0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
Partitioning	1	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
					Model 3.2									
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.407	0.430	0.317	0.335	0.435	0.462	0.395	0.420	0.462	0.487	0.472	0.487
B-splines	27	8	0.403	0.421	0.317	0.327	0.401	0.463	0.347	0.414	0.437	0.483	0.407	0.490
Partitioning	27	8	0.491	0.575	0.383	0.443	0.345	2.468	0.440	2.176	0.539	1.702	0.621	1.288
Feasible Estimation														
Local Polynomial	0.37	0.27	0.374	0.384	0.286	0.296	0.374	0.380	0.312	0.327	0.300	0.308	0.294	0.309
B-splines	2	1	0.377	0.402	0.289	0.314	0.381	0.408	0.302	0.351	0.333	0.450	0.308	0.433
Partitioning	2	1	0.379	0.404	0.291	0.315	0.401	0.411	0.337	0.481	0.337	0.368	0.343	0.484
					Model 3.3									
Infeasible Estimation														
Local Polynomial	0.28	0.34	0.168	0.231	0.120	0.171	0.088	0.132	0.164	0.197	0.282	0.323	0.282	0.323
B-splines	8	1	0.156	0.195	0.114	0.142	0.165	0.105	0.192	0.178	0.234	0.331	0.234	0.331
Partitioning	8	1	0.257	0.206	0.201	0.155	0.424	0.105	0.373	0.184	0.358	0.284	0.348	0.477
Feasible Estimation														
Local Polynomial	0.38	0.27	0.123	0.156	0.077	0.109	0.056	0.093	0.114	0.142	0.114	0.142	0.114	0.142
B-splines	1	1	0.126	0.195	0.085	0.142	0.058	0.105	0.086	0.178	0.153	0.331	0.153	0.331
Partitioning	1	1	0.131	0.206	0.090	0.155	0.104	0.105	0.104	0.184	0.129	0.284	0.173	0.477
					Model 3.4									
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.785	1.320	0.508	0.496	0.331	0.148	0.413	0.353	7.493	0.619	8.533	3.831
B-splines	216	27	1.224	1.175	0.570	0.539	0.389	0.162	0.520	0.459	2.179	1.985	4.443	4.338
Partitioning	216	27	1.281	1.214	0.470	0.536	2.240	1.419	0.948	3.605	1.165	4.904	3.797	8.345
Feasible Estimation														
Local Polynomial	0.29	0.26	1.345	1.327	0.469	0.489	0.063	0.096	0.431	0.325	0.431	0.325	3.805	3.800
B-splines	8	2	1.333	1.271	0.524	0.533	0.204	0.145	0.350	0.408	0.799	1.625	3.872	4.147
Partitioning	8	2	1.114	1.112	0.645	0.601	1.328	1.213	0.522	0.971	1.123	0.974	2.218	2.807
					Model 3.5									
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.789	0.613	0.616	0.493	1.130	1.205	1.151	1.052	0.716	0.734	0.622	0.641
B-splines	27	27	0.848	0.635	0.668	0.510	0.928	1.341	1.003	1.207	0.893	0.787	0.541	0.799
Partitioning	27	27	0.738	1.279	0.576	1.013	0.790	1.478	0.789	4.460	0.658	5.563	0.762	7.308
Feasible Estimation														
Local Polynomial	0.28	0.28	0.750	0.593	0.587	0.477	1.216	1.225	1.262	1.173	0.580	0.669	0.598	0.514
B-splines	8	1	0.890	0.805	0.688	0.642	0.725	0.814	0.972	0.980	0.826	0.944	0.485	0.596
Partitioning	8	1	0.852	0.786	0.655	0.623	0.962	0.913	1.110	1.071	0.936	0.762	0.779	0.822

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.63: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2,2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.356	0.366	0.298	0.304	0.048	0.083	0.104	0.190	0.151	0.277	0.655	0.697
B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
Feasible Estimation														
Local Polynomial	0.43	0.29	0.497	0.504	0.404	0.409	0.053	0.087	0.164	0.226	0.662	0.679	1.291	1.300
B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.18	0.25	0.401	0.419	0.305	0.320	0.365	0.369	0.428	0.413	1.556	1.224	1.557	1.240
B-splines	27	8	0.398	0.416	0.307	0.317	0.365	0.369	0.355	0.450	0.584	1.350	0.637	1.330
Partitioning	27	8	0.491	0.571	0.382	0.435	0.309	1.914	0.571	2.271	0.839	2.714	0.980	3.852
Feasible Estimation														
Local Polynomial	0.36	0.26	0.369	0.380	0.278	0.288	0.381	0.385	0.345	0.369	0.309	0.338	0.328	0.353
B-splines	4	4	0.376	0.405	0.285	0.310	0.373	0.367	0.326	0.414	0.382	1.092	0.435	1.092
Partitioning	4	4	0.380	0.479	0.291	0.361	0.466	1.308	0.444	1.500	0.502	1.778	0.725	2.676
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.34	0.36	0.138	0.195	0.095	0.138	0.060	0.097	0.182	0.237	0.399	0.549	0.399	0.549
B-splines	1	1	0.106	0.186	0.074	0.131	0.046	0.087	0.091	0.240	0.197	0.754	0.197	0.754
Partitioning	1	1	0.107	0.203	0.077	0.149	0.046	0.088	0.091	0.221	0.134	0.426	0.169	0.745
Feasible Estimation														
Local Polynomial	0.37	0.27	0.100	0.137	0.066	0.095	0.051	0.081	0.157	0.203	0.157	0.203	0.157	0.203
B-splines	2	1	0.117	0.187	0.081	0.131	0.079	0.088	0.135	0.243	0.253	0.770	0.253	0.770
Partitioning	2	1	0.152	0.213	0.102	0.153	0.157	0.267	0.187	0.383	0.227	0.507	0.257	0.820
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.11	0.9	0.588	0.551	0.256	0.208	0.166	0.095	0.473	0.339	4.660	0.507	5.969	3.837
B-splines	64	27	0.553	0.485	0.273	0.269	0.253	0.128	0.453	0.490	1.508	3.629	4.066	5.147
Partitioning	64	27	0.735	0.721	0.551	0.356	1.848	2.783	1.741	10.693	2.392	17.645	3.636	34.300
Feasible Estimation														
Local Polynomial	0.32	0.26	0.561	0.544	0.184	0.202	0.053	0.082	0.361	0.415	0.361	0.415	3.818	3.821
B-splines	7	3	0.561	0.514	0.222	0.242	0.146	0.113	0.268	0.396	0.672	2.145	3.864	4.330
Partitioning	7	3	0.543	0.524	0.298	0.310	0.538	1.083	0.426	1.224	0.656	1.560	3.255	3.847
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.720	0.611	0.560	0.493	1.090	1.058	0.831	0.952	1.026	0.858	0.623	0.958
B-splines	27	27	0.775	0.646	0.598	0.517	0.961	1.175	0.695	0.930	1.149	3.559	0.714	3.221
Partitioning	27	27	0.699	1.127	0.536	0.914	0.606	2.796	0.698	9.955	0.937	17.105	1.039	37.494
Feasible Estimation														
Local Polynomial	0.28	0.27	0.713	0.616	0.558	0.495	1.199	1.194	0.984	1.178	0.882	0.697	0.362	0.534
B-splines	8	4	0.775	0.705	0.592	0.560	0.805	1.055	0.637	0.830	0.977	1.698	0.526	1.391
Partitioning	8	4	0.725	0.669	0.542	0.517	0.947	1.618	0.777	1.591	1.172	2.182	1.069	3.256

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.64: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.484	0.516	0.417	0.434	0.094	0.185	0.165	0.246	0.242	0.304	0.693	0.715
B-splines	1	1	0.488	0.573	0.419	0.470	0.093	0.235	0.157	0.340	0.281	0.486	0.709	0.809
Partitioning	1	1	0.179	0.399	0.141	0.307	0.090	0.206	0.151	0.345	0.199	0.475	0.242	0.671
Feasible Estimation														
Local Polynomial	0.38	0.28	0.662	0.686	0.540	0.558	0.122	0.213	0.213	0.270	0.676	0.695	1.299	1.309
B-splines	1	1	0.489	0.573	0.420	0.470	0.103	0.235	0.167	0.340	0.284	0.486	0.710	0.809
Partitioning	1	1	0.190	0.399	0.145	0.307	0.148	0.206	0.191	0.345	0.210	0.475	0.250	0.671
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.497	0.583	0.391	0.456	0.505	0.590	0.531	0.616	0.589	0.643	0.535	0.563
B-splines	8	8	0.447	0.566	0.350	0.441	0.573	0.617	0.544	0.615	0.544	0.601	0.427	0.539
Partitioning	8	8	0.594	1.136	0.467	0.875	0.940	6.140	0.940	4.325	0.782	2.587	0.630	1.496
Feasible Estimation														
Local Polynomial	0.34	0.26	0.398	0.438	0.309	0.343	0.373	0.405	0.342	0.378	0.357	0.388	0.327	0.364
B-splines	4	1	0.425	0.504	0.331	0.397	0.468	0.460	0.422	0.468	0.488	0.592	0.387	0.513
Partitioning	4	1	0.495	0.532	0.380	0.418	0.694	0.800	0.677	0.699	0.601	0.545	0.462	0.685
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.37	0.303	0.426	0.227	0.326	0.192	0.287	0.316	0.372	0.450	0.472	0.450	0.472
B-splines	1	1	0.208	0.370	0.154	0.281	0.097	0.232	0.151	0.331	0.271	0.473	0.271	0.473
Partitioning	1	1	0.209	0.403	0.158	0.310	0.097	0.207	0.151	0.346	0.206	0.477	0.261	0.682
Feasible Estimation														
Local Polynomial	0.35	0.26	0.199	0.273	0.143	0.207	0.124	0.208	0.204	0.259	0.204	0.259	0.204	0.259
B-splines	3	1	0.240	0.370	0.177	0.281	0.225	0.232	0.255	0.333	0.318	0.473	0.318	0.473
Partitioning	3	1	0.348	0.407	0.240	0.311	0.497	0.348	0.475	0.442	0.442	0.491	0.400	0.683
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.9	1.872	2.055	1.007	0.891	0.771	0.234	0.735	0.413	1.504	0.726	4.087	3.835
B-splines	125	27	2.004	1.947	1.006	0.990	0.315	0.344	0.551	0.580	1.690	1.412	4.168	4.037
Partitioning	125	27	1.878	2.010	1.076	1.024	1.971	3.386	2.111	4.385	4.015	5.901	3.903	6.906
Feasible Estimation														
Local Polynomial	0.3	0.26	2.083	2.070	0.846	0.882	0.135	0.211	0.506	0.408	0.506	0.408	3.795	3.793
B-splines	8	1	2.074	2.028	0.925	0.946	0.408	0.272	0.513	0.534	0.898	1.407	3.872	4.041
Partitioning	8	1	1.658	1.772	1.080	1.069	2.360	2.135	0.897	1.257	1.664	0.951	1.539	2.408
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.852	0.715	0.666	0.569	1.145	1.303	1.331	1.202	0.720	0.924	0.739	0.686
B-splines	27	27	0.900	0.769	0.719	0.609	0.888	1.405	1.216	1.342	0.869	0.852	0.683	0.741
Partitioning	27	27	1.072	1.697	0.840	1.359	1.075	3.397	1.230	4.503	0.918	6.327	1.043	6.370
Feasible Estimation														
Local Polynomial	0.28	0.26	0.783	0.605	0.614	0.486	1.200	1.214	1.364	1.184	0.514	0.709	0.706	0.547
B-splines	8	1	0.969	0.878	0.759	0.703	0.739	0.740	1.210	1.202	0.849	0.885	0.591	0.678
Partitioning	8	1	1.003	0.872	0.789	0.696	1.183	0.887	1.435	1.354	1.009	0.720	0.783	0.834

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.65: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.443	0.480	0.375	0.398	0.089	0.179	0.170	0.267	0.244	0.361	0.688	0.735
B-splines	1	1	0.449	0.542	0.379	0.437	0.088	0.212	0.160	0.367	0.305	0.682	0.708	0.899
Partitioning	1	1	0.174	0.399	0.136	0.301	0.086	0.204	0.158	0.361	0.207	0.567	0.249	0.874
Feasible Estimation														
Local Polynomial	0.38	0.27	0.606	0.633	0.494	0.514	0.112	0.192	0.239	0.297	0.687	0.710	1.306	1.319
B-splines	1	1	0.450	0.542	0.380	0.437	0.101	0.212	0.169	0.367	0.310	0.682	0.710	0.899
Partitioning	1	1	0.194	0.399	0.144	0.301	0.175	0.204	0.206	0.361	0.250	0.567	0.276	0.874
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.501	0.582	0.389	0.450	0.466	0.533	0.502	0.542	0.705	0.719	0.724	0.729
B-splines	8	8	0.450	0.572	0.350	0.441	0.543	0.559	0.491	0.583	0.563	0.812	0.534	0.837
Partitioning	8	8	0.595	1.136	0.467	0.873	0.916	4.605	0.888	4.141	0.841	3.269	0.852	2.457
Feasible Estimation														
Local Polynomial	0.34	0.25	0.399	0.439	0.309	0.343	0.387	0.416	0.373	0.410	0.363	0.396	0.358	0.396
B-splines	5	1	0.432	0.510	0.335	0.398	0.479	0.456	0.422	0.470	0.509	0.724	0.486	0.732
Partitioning	5	1	0.518	0.571	0.397	0.435	0.711	1.039	0.655	0.943	0.679	0.880	0.677	0.967
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.34	0.38	0.277	0.395	0.201	0.296	0.146	0.232	0.294	0.346	0.532	0.555	0.532	0.555
B-splines	1	1	0.195	0.365	0.143	0.270	0.090	0.209	0.159	0.355	0.298	0.662	0.298	0.662
Partitioning	1	1	0.196	0.402	0.148	0.304	0.090	0.205	0.158	0.363	0.213	0.567	0.269	0.899
Feasible Estimation														
Local Polynomial	0.34	0.25	0.185	0.264	0.133	0.197	0.116	0.192	0.234	0.285	0.234	0.285	0.234	0.285
B-splines	4	1	0.245	0.367	0.179	0.271	0.238	0.215	0.302	0.362	0.405	0.666	0.405	0.666
Partitioning	4	1	0.389	0.428	0.274	0.314	0.582	0.547	0.518	0.680	0.543	0.711	0.553	0.941
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.1	0.9	1.209	1.337	0.676	0.550	0.517	0.215	0.653	0.420	2.118	0.693	4.419	3.844
B-splines	125	27	1.337	1.263	0.708	0.676	0.309	0.307	0.563	0.603	2.095	2.143	4.420	4.439
Partitioning	125	27	1.531	1.559	0.910	0.805	1.702	2.838	1.778	7.185	1.861	9.825	4.003	15.041
Feasible Estimation														
Local Polynomial	0.3	0.25	1.353	1.342	0.500	0.539	0.123	0.193	0.475	0.413	0.475	0.413	3.809	3.807
B-splines	8	3	1.353	1.310	0.576	0.616	0.350	0.268	0.486	0.528	0.891	1.758	3.899	4.221
Partitioning	8	3	1.198	1.219	0.745	0.735	1.499	2.596	0.824	1.992	1.273	1.663	2.326	3.068
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.822	0.723	0.641	0.575	1.139	1.228	1.171	1.096	0.843	0.918	0.783	0.872
B-splines	27	27	0.903	0.787	0.711	0.622	0.944	1.367	1.031	1.262	1.023	1.087	0.760	1.127
Partitioning	27	27	1.089	1.610	0.855	1.282	0.932	2.873	1.069	7.927	1.116	10.236	1.325	14.255
Feasible Estimation														
Local Polynomial	0.29	0.26	0.774	0.624	0.605	0.501	1.212	1.220	1.265	1.205	0.629	0.708	0.619	0.575
B-splines	8	1	0.922	0.860	0.711	0.682	0.812	0.864	0.994	1.023	0.933	1.080	0.636	0.839
Partitioning	8	1	0.958	0.869	0.743	0.684	1.179	1.319	1.252	1.204	1.137	1.058	0.988	1.237

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.66: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 500$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2, 2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.383	0.423	0.316	0.344	0.091	0.159	0.197	0.363	0.290	0.529	0.699	0.828
B-splines	1	1	0.391	0.496	0.321	0.388	0.090	0.177	0.185	0.489	0.396	1.550	0.752	1.685
Partitioning	1	1	0.180	0.401	0.139	0.294	0.089	0.175	0.180	0.438	0.253	0.851	0.303	1.409
Feasible Estimation														
Local Polynomial	0.37	0.27	0.516	0.547	0.418	0.440	0.103	0.166	0.327	0.422	0.719	0.768	1.322	1.349
B-splines	2	1	0.401	0.497	0.328	0.389	0.149	0.179	0.267	0.496	0.496	1.579	0.811	1.719
Partitioning	2	1	0.275	0.421	0.185	0.302	0.292	0.520	0.361	0.754	0.421	1.007	0.497	1.568
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.492	0.594	0.371	0.441	0.391	0.416	0.594	0.635	1.385	1.672	1.387	1.681
B-splines	8	8	0.447	0.568	0.342	0.426	0.433	0.442	0.512	0.755	0.731	2.521	0.789	2.495
Partitioning	8	8	0.586	1.133	0.456	0.863	0.834	3.674	0.874	4.349	1.010	5.289	1.297	7.269
Feasible Estimation														
Local Polynomial	0.33	0.25	0.396	0.437	0.302	0.334	0.392	0.409	0.450	0.519	0.423	0.500	0.437	0.508
B-splines	7	6	0.441	0.552	0.337	0.415	0.421	0.431	0.494	0.712	0.703	2.280	0.754	2.265
Partitioning	7	6	0.561	1.008	0.432	0.740	0.751	3.217	0.812	3.671	0.929	4.657	1.204	6.238
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.41	0.41	0.231	0.346	0.165	0.248	0.105	0.177	0.305	0.442	0.625	0.956	0.625	0.956
B-splines	1	1	0.188	0.361	0.138	0.256	0.089	0.174	0.181	0.480	0.392	1.509	0.392	1.509
Partitioning	1	1	0.189	0.402	0.144	0.295	0.089	0.175	0.181	0.439	0.257	0.851	0.313	1.431
Feasible Estimation														
Local Polynomial	0.33	0.25	0.173	0.255	0.123	0.182	0.105	0.166	0.329	0.417	0.329	0.417	0.329	0.417
B-splines	6	5	0.261	0.413	0.190	0.291	0.246	0.246	0.411	0.623	0.645	2.143	0.645	2.143
Partitioning	6	5	0.460	0.881	0.340	0.604	0.605	2.794	0.688	3.117	0.764	3.925	0.803	5.428
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.13	0.9	0.632	0.591	0.387	0.277	0.267	0.166	0.746	0.453	4.138	0.672	5.652	3.864
B-splines	64	27	0.674	0.671	0.412	0.434	0.504	0.251	0.831	0.801	1.853	5.010	4.186	6.171
Partitioning	64	27	1.390	1.207	1.057	0.621	3.697	5.566	3.478	21.390	4.558	35.146	5.091	68.472
Feasible Estimation														
Local Polynomial	0.31	0.25	0.579	0.582	0.224	0.267	0.106	0.167	0.459	0.544	0.459	0.544	3.828	3.835
B-splines	8	6	0.606	0.627	0.295	0.370	0.278	0.262	0.464	0.720	0.890	3.030	3.910	4.752
Partitioning	8	6	0.692	1.019	0.459	0.727	0.801	3.201	0.785	3.659	0.980	4.489	3.325	6.612
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.755	0.705	0.585	0.560	1.095	1.071	0.892	1.043	1.250	1.368	0.944	1.438
B-splines	27	27	0.834	0.795	0.644	0.620	0.971	1.193	0.780	1.124	1.443	5.097	1.123	4.857
Partitioning	27	27	1.066	1.486	0.830	1.180	0.781	5.568	1.196	20.552	1.588	34.372	1.812	70.198
Feasible Estimation														
Local Polynomial	0.29	0.25	0.733	0.646	0.572	0.517	1.197	1.186	1.011	1.236	0.944	0.782	0.455	0.648
B-splines	8	6	0.810	0.778	0.617	0.612	0.847	1.150	0.740	1.073	1.151	2.637	0.797	2.461
Partitioning	8	6	0.846	1.073	0.643	0.814	1.114	3.262	0.993	3.600	1.382	4.737	1.311	6.698

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.67: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.458	0.462	0.403	0.404	0.034	0.073	0.062	0.092	0.091	0.116	0.648	0.656
B-splines	1	1	0.459	0.469	0.404	0.407	0.034	0.093	0.060	0.124	0.103	0.177	0.650	0.667
Partitioning	1	1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
Feasible Estimation														
Local Polynomial	0.46	0.31	0.645	0.648	0.526	0.528	0.043	0.081	0.080	0.106	0.648	0.653	1.285	1.288
B-splines	1	1	0.459	0.469	0.404	0.407	0.034	0.093	0.060	0.124	0.103	0.177	0.650	0.667
Partitioning	1	1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.382	0.396	0.297	0.307	0.468	0.497	0.392	0.432	0.370	0.402	0.287	0.319
B-splines	27	8	0.380	0.386	0.296	0.297	0.394	0.494	0.327	0.405	0.383	0.377	0.263	0.268
Partitioning	27	8	0.378	0.421	0.296	0.322	0.309	1.913	0.302	1.287	0.285	0.774	0.332	0.420
Feasible Estimation														
Local Polynomial	0.38	0.28	0.369	0.374	0.281	0.287	0.357	0.353	0.284	0.293	0.298	0.302	0.265	0.274
B-splines	1	1	0.370	0.380	0.283	0.294	0.377	0.408	0.270	0.337	0.340	0.381	0.246	0.261
Partitioning	1	1	0.370	0.375	0.284	0.291	0.387	0.394	0.300	0.534	0.332	0.280	0.263	0.249
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.22	0.3	0.156	0.202	0.112	0.151	0.095	0.127	0.131	0.170	0.166	0.216	0.166	0.216
B-splines	8	8	0.138	0.186	0.096	0.138	0.139	0.151	0.136	0.158	0.135	0.166	0.135	0.166
Partitioning	8	8	0.187	0.400	0.146	0.305	0.342	1.529	0.252	1.064	0.241	0.719	0.198	0.411
Feasible Estimation														
Local Polynomial	0.39	0.28	0.130	0.146	0.074	0.094	0.046	0.072	0.072	0.091	0.072	0.091	0.072	0.091
B-splines	1	1	0.125	0.162	0.080	0.116	0.049	0.085	0.057	0.114	0.096	0.161	0.096	0.161
Partitioning	1	1	0.126	0.153	0.081	0.117	0.061	0.083	0.059	0.122	0.099	0.171	0.142	0.255
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.07	0.9	1.844	2.068	0.813	0.823	0.361	0.133	0.275	0.237	0.945	0.472	3.936	3.837
B-splines	343	64	1.972	1.948	0.852	0.849	0.159	0.202	0.259	0.297	0.972	0.782	3.939	3.888
Partitioning	343	64	1.795	1.982	0.743	0.771	0.599	49.860	0.497	7.261	0.687	4.008	3.591	3.773
Feasible Estimation														
Local Polynomial	0.29	0.26	2.084	2.076	0.803	0.816	0.049	0.077	0.317	0.226	0.317	0.226	3.817	3.800
B-splines	8	2	2.076	2.047	0.846	0.848	0.181	0.119	0.288	0.310	0.605	0.970	3.864	3.951
Partitioning	8	2	1.618	1.740	1.022	0.984	2.244	1.327	0.415	0.474	1.442	0.550	1.318	2.094
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.818	0.582	0.639	0.465	1.141	1.285	1.306	1.119	0.578	0.747	0.619	0.468
B-splines	27	27	0.845	0.589	0.677	0.471	0.871	1.391	1.187	1.264	0.716	0.658	0.505	0.544
Partitioning	27	27	0.665	0.743	0.520	0.579	0.933	0.677	1.002	0.806	0.328	0.779	0.423	1.042
Feasible Estimation														
Local Polynomial	0.25	0.28	0.725	0.567	0.570	0.455	1.212	1.213	1.385	1.156	0.438	0.667	0.712	0.487
B-splines	8	1	0.936	0.820	0.735	0.661	0.587	0.711	1.158	1.166	0.717	0.724	0.473	0.501
Partitioning	8	1	0.898	0.797	0.704	0.639	0.888	0.862	1.314	1.321	0.738	0.534	0.589	0.532

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.68: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.419	0.423	0.362	0.364	0.036	0.065	0.067	0.104	0.095	0.135	0.646	0.649
B-splines	1	1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
Feasible Estimation														
Local Polynomial	0.45	0.31	0.590	0.592	0.481	0.484	0.042	0.073	0.094	0.119	0.648	0.653	1.285	1.287
B-splines	1	1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.390	0.403	0.303	0.312	0.424	0.442	0.368	0.391	0.386	0.412	0.366	0.386
B-splines	27	8	0.386	0.394	0.300	0.305	0.390	0.437	0.330	0.379	0.374	0.383	0.329	0.357
Partitioning	27	8	0.374	0.420	0.293	0.321	0.266	1.709	0.295	1.304	0.343	0.940	0.403	0.699
Feasible Estimation														
Local Polynomial	0.37	0.27	0.372	0.377	0.283	0.288	0.370	0.373	0.301	0.309	0.289	0.294	0.283	0.291
B-splines	2	1	0.373	0.385	0.285	0.298	0.379	0.393	0.303	0.333	0.314	0.381	0.287	0.342
Partitioning	2	1	0.373	0.383	0.285	0.296	0.397	0.402	0.333	0.469	0.316	0.302	0.325	0.346
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.25	0.32	0.139	0.183	0.097	0.133	0.069	0.098	0.130	0.155	0.208	0.236	0.208	0.236
B-splines	8	1	0.126	0.151	0.088	0.107	0.115	0.071	0.139	0.126	0.165	0.229	0.165	0.229
Partitioning	8	1	0.186	0.150	0.145	0.113	0.295	0.072	0.271	0.131	0.249	0.199	0.225	0.335
Feasible Estimation														
Local Polynomial	0.38	0.28	0.112	0.130	0.065	0.085	0.043	0.067	0.085	0.103	0.085	0.103	0.085	0.103
B-splines	1	1	0.111	0.151	0.072	0.107	0.045	0.071	0.068	0.126	0.114	0.229	0.114	0.229
Partitioning	1	1	0.115	0.150	0.075	0.113	0.073	0.072	0.076	0.131	0.106	0.199	0.142	0.335
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.284	1.342	0.491	0.472	0.235	0.096	0.265	0.254	1.381	0.447	4.003	3.809
B-splines	216	27	1.292	1.263	0.529	0.511	0.284	0.115	0.364	0.322	1.539	1.348	4.064	3.989
Partitioning	216	27	1.040	0.749	0.582	0.584	2.265	0.582	2.102	0.920	1.354	1.951	3.390	2.491
Feasible Estimation														
Local Polynomial	0.29	0.26	1.355	1.346	0.454	0.468	0.046	0.069	0.316	0.237	0.316	0.237	3.802	3.790
B-splines	8	3	1.349	1.313	0.493	0.501	0.146	0.104	0.254	0.310	0.587	1.198	3.820	3.942
Partitioning	8	3	1.127	1.083	0.640	0.557	1.295	1.460	0.420	0.634	1.052	0.741	2.136	2.513
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.789	0.601	0.616	0.485	1.125	1.200	1.147	1.047	0.663	0.680	0.570	0.567
B-splines	27	27	0.848	0.616	0.668	0.496	0.919	1.339	0.995	1.204	0.836	0.617	0.447	0.658
Partitioning	27	27	0.700	0.747	0.544	0.584	0.773	0.628	0.720	0.937	0.446	1.831	0.479	2.136
Feasible Estimation														
Local Polynomial	0.25	0.28	0.720	0.589	0.566	0.474	1.225	1.225	1.286	1.166	0.540	0.662	0.616	0.499
B-splines	8	1	0.890	0.802	0.688	0.641	0.683	0.808	0.967	0.968	0.793	0.854	0.430	0.475
Partitioning	8	1	0.847	0.780	0.650	0.619	0.914	0.923	1.066	1.068	0.860	0.729	0.704	0.665

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.69: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 1$, $X_{i,\ell} \sim \beta(2,2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.353	0.358	0.296	0.299	0.033	0.055	0.071	0.134	0.101	0.188	0.648	0.669
B-splines	1	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
Feasible Estimation														
Local Polynomial	0.43	0.3	0.495	0.499	0.403	0.406	0.037	0.060	0.113	0.155	0.655	0.669	1.290	1.301
B-splines	1	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.384	0.393	0.292	0.299	0.352	0.351	0.363	0.350	0.722	0.553	0.743	0.590
B-splines	27	8	0.379	0.389	0.290	0.296	0.357	0.350	0.317	0.364	0.424	0.714	0.500	0.745
Partitioning	27	8	0.372	0.413	0.290	0.311	0.210	1.118	0.361	1.408	0.559	1.598	0.671	2.076
Feasible Estimation														
Local Polynomial	0.36	0.27	0.366	0.371	0.274	0.279	0.376	0.378	0.320	0.328	0.293	0.310	0.302	0.311
B-splines	4	4	0.369	0.384	0.277	0.292	0.360	0.353	0.300	0.352	0.320	0.644	0.374	0.682
Partitioning	4	4	0.360	0.397	0.274	0.300	0.437	0.827	0.433	1.060	0.470	1.203	0.706	1.715
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.31	0.34	0.110	0.150	0.074	0.105	0.046	0.074	0.143	0.175	0.319	0.350	0.319	0.350
B-splines	8	1	0.108	0.136	0.076	0.094	0.104	0.061	0.160	0.173	0.239	0.470	0.239	0.470
Partitioning	8	1	0.181	0.145	0.140	0.105	0.258	0.062	0.277	0.154	0.278	0.281	0.305	0.525
Feasible Estimation														
Local Polynomial	0.37	0.27	0.083	0.106	0.051	0.070	0.037	0.057	0.115	0.142	0.115	0.142	0.115	0.142
B-splines	2	1	0.091	0.136	0.061	0.094	0.054	0.061	0.091	0.173	0.165	0.470	0.165	0.470
Partitioning	2	1	0.112	0.145	0.074	0.105	0.124	0.062	0.132	0.154	0.149	0.281	0.180	0.525
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.1	0.9	0.485	0.554	0.222	0.183	0.137	0.065	0.331	0.248	2.492	0.368	4.498	3.820
B-splines	125	27	0.550	0.497	0.249	0.230	0.107	0.096	0.263	0.348	1.442	2.209	4.072	4.394
Partitioning	125	27	0.723	0.738	0.545	0.574	0.772	0.616	1.332	2.296	1.774	4.846	3.596	10.280
Feasible Estimation														
Local Polynomial	0.31	0.26	0.560	0.551	0.168	0.182	0.038	0.058	0.265	0.297	0.265	0.297	3.799	3.798
B-splines	8	3	0.560	0.525	0.195	0.212	0.114	0.084	0.212	0.299	0.498	1.634	3.833	4.145
Partitioning	8	3	0.525	0.483	0.263	0.256	0.470	0.739	0.328	0.792	0.564	1.105	3.153	3.158
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.719	0.602	0.559	0.487	1.087	1.049	0.812	0.924	0.896	0.657	0.498	0.752
B-splines	27	27	0.773	0.629	0.596	0.507	0.954	1.172	0.669	0.914	0.990	1.450	0.517	1.229
Partitioning	27	27	0.652	0.748	0.489	0.584	0.561	0.625	0.456	2.292	0.666	4.910	0.674	10.386
Feasible Estimation														
Local Polynomial	0.26	0.27	0.696	0.612	0.548	0.492	1.207	1.192	1.014	1.157	0.823	0.685	0.361	0.498
B-splines	8	7	0.773	0.663	0.591	0.532	0.774	1.160	0.603	0.937	0.915	1.026	0.413	0.926
Partitioning	8	7	0.715	0.512	0.532	0.392	0.930	1.270	0.759	1.384	1.119	1.502	0.991	1.949

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.70: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(0.5, 0.5)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.469	0.487	0.409	0.418	0.063	0.136	0.117	0.172	0.171	0.218	0.665	0.685
B-splines	1	1	0.471	0.518	0.410	0.436	0.063	0.174	0.111	0.232	0.195	0.330	0.669	0.725
Partitioning	1	1	0.125	0.283	0.098	0.217	0.060	0.154	0.108	0.239	0.143	0.337	0.170	0.451
Feasible Estimation														
Local Polynomial	0.39	0.28	0.652	0.665	0.532	0.542	0.084	0.153	0.148	0.192	0.661	0.672	1.293	1.299
B-splines	1	1	0.472	0.518	0.411	0.436	0.071	0.174	0.116	0.232	0.195	0.330	0.669	0.725
Partitioning	1	1	0.132	0.283	0.101	0.217	0.110	0.154	0.124	0.239	0.155	0.337	0.173	0.451
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.443	0.493	0.348	0.386	0.491	0.548	0.447	0.512	0.477	0.509	0.409	0.444
B-splines	27	8	0.439	0.474	0.349	0.370	0.414	0.559	0.371	0.485	0.479	0.476	0.386	0.390
Partitioning	27	8	0.684	0.810	0.535	0.619	0.428	3.265	0.503	2.244	0.552	1.468	0.657	0.825
Feasible Estimation														
Local Polynomial	0.35	0.26	0.381	0.403	0.294	0.314	0.362	0.378	0.309	0.334	0.325	0.342	0.292	0.318
B-splines	4	1	0.395	0.438	0.306	0.345	0.442	0.437	0.373	0.387	0.416	0.474	0.311	0.380
Partitioning	4	1	0.429	0.449	0.332	0.353	0.577	0.420	0.558	0.614	0.471	0.408	0.377	0.463
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.27	0.34	0.235	0.325	0.175	0.248	0.152	0.219	0.229	0.283	0.309	0.355	0.309	0.355
B-splines	8	1	0.215	0.272	0.162	0.205	0.277	0.170	0.272	0.228	0.269	0.323	0.269	0.323
Partitioning	8	1	0.362	0.289	0.283	0.222	0.615	0.157	0.504	0.240	0.482	0.338	0.397	0.465
Feasible Estimation														
Local Polynomial	0.35	0.26	0.163	0.210	0.110	0.155	0.088	0.152	0.146	0.186	0.146	0.186	0.146	0.186
B-splines	3	1	0.184	0.272	0.133	0.205	0.177	0.170	0.189	0.228	0.223	0.323	0.223	0.323
Partitioning	3	1	0.251	0.289	0.174	0.222	0.377	0.157	0.306	0.240	0.309	0.338	0.288	0.465
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.08	0.9	1.957	2.074	0.956	0.852	0.552	0.175	0.457	0.278	1.047	0.507	3.965	3.837
B-splines	216	27	2.035	2.018	0.960	0.931	0.563	0.254	0.509	0.394	1.125	0.949	3.999	3.943
Partitioning	216	27	1.780	1.479	1.105	1.152	4.550	1.170	5.118	1.460	2.295	1.531	3.443	2.271
Feasible Estimation														
Local Polynomial	0.3	0.25	2.087	2.081	0.819	0.844	0.095	0.152	0.340	0.281	0.340	0.281	3.816	3.805
B-splines	8	2	2.083	2.059	0.873	0.892	0.301	0.201	0.376	0.375	0.652	1.006	3.873	3.958
Partitioning	8	2	1.647	1.767	1.053	1.023	2.302	1.373	0.597	0.731	1.502	0.722	1.353	2.147
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.835	0.645	0.652	0.515	1.147	1.302	1.314	1.145	0.636	0.812	0.668	0.565
B-splines	27	27	0.873	0.675	0.699	0.538	0.881	1.408	1.197	1.284	0.773	0.723	0.577	0.620
Partitioning	27	27	0.876	1.473	0.689	1.147	0.976	1.219	1.079	1.497	0.573	1.534	0.711	2.081
Feasible Estimation														
Local Polynomial	0.26	0.26	0.746	0.581	0.585	0.467	1.211	1.204	1.374	1.169	0.471	0.684	0.708	0.514
B-splines	8	1	0.951	0.849	0.746	0.681	0.638	0.732	1.179	1.179	0.753	0.777	0.527	0.571
Partitioning	8	1	0.950	0.834	0.747	0.668	1.024	0.877	1.382	1.350	0.855	0.609	0.677	0.658

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.71: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(1,1)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.432	0.451	0.369	0.380	0.067	0.122	0.126	0.195	0.180	0.254	0.663	0.682
B-splines	1	1	0.435	0.484	0.371	0.401	0.067	0.144	0.120	0.257	0.221	0.465	0.680	0.781
Partitioning	1	1	0.129	0.284	0.101	0.215	0.066	0.137	0.118	0.254	0.155	0.394	0.180	0.575
Feasible Estimation														
Local Polynomial	0.38	0.28	0.598	0.612	0.488	0.498	0.083	0.139	0.178	0.218	0.669	0.682	1.297	1.304
B-splines	1	1	0.436	0.484	0.372	0.401	0.077	0.144	0.131	0.257	0.226	0.465	0.681	0.781
Partitioning	1	1	0.144	0.284	0.107	0.215	0.131	0.137	0.144	0.254	0.182	0.394	0.193	0.575
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.450	0.496	0.351	0.386	0.438	0.480	0.440	0.474	0.553	0.558	0.541	0.544
B-splines	27	8	0.444	0.480	0.352	0.373	0.402	0.484	0.383	0.484	0.522	0.568	0.495	0.555
Partitioning	27	8	0.681	0.810	0.532	0.617	0.414	2.934	0.543	2.282	0.673	1.756	0.795	1.300
Feasible Estimation														
Local Polynomial	0.34	0.26	0.385	0.407	0.297	0.316	0.377	0.393	0.335	0.354	0.329	0.348	0.318	0.338
B-splines	5	1	0.403	0.444	0.311	0.348	0.429	0.411	0.389	0.398	0.425	0.548	0.383	0.525
Partitioning	5	1	0.442	0.461	0.343	0.360	0.593	0.525	0.593	0.594	0.514	0.499	0.534	0.624
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.36	0.218	0.301	0.158	0.224	0.113	0.171	0.233	0.267	0.401	0.405	0.401	0.405
B-splines	1	1	0.157	0.266	0.114	0.197	0.071	0.141	0.119	0.252	0.216	0.459	0.216	0.459
Partitioning	1	1	0.157	0.288	0.116	0.218	0.071	0.139	0.119	0.257	0.166	0.395	0.209	0.604
Feasible Estimation														
Local Polynomial	0.34	0.26	0.151	0.201	0.105	0.147	0.085	0.138	0.174	0.208	0.174	0.208	0.174	0.208
B-splines	4	1	0.187	0.266	0.136	0.197	0.173	0.141	0.223	0.253	0.283	0.459	0.283	0.459
Partitioning	4	1	0.280	0.293	0.198	0.219	0.398	0.270	0.391	0.322	0.367	0.417	0.353	0.612
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.9	1.255	1.351	0.615	0.508	0.379	0.139	0.451	0.302	1.481	0.496	4.046	3.817
B-splines	125	27	1.348	1.305	0.618	0.601	0.213	0.218	0.401	0.431	1.494	1.429	4.056	4.020
Partitioning	125	27	1.438	1.477	1.091	1.150	1.635	1.163	1.799	1.833	2.576	3.885	3.695	4.473
Feasible Estimation														
Local Polynomial	0.3	0.26	1.359	1.354	0.475	0.501	0.090	0.139	0.348	0.300	0.348	0.300	3.801	3.790
B-splines	8	3	1.359	1.334	0.527	0.555	0.250	0.181	0.353	0.390	0.656	1.257	3.831	3.967
Partitioning	8	3	1.169	1.171	0.692	0.648	1.382	1.786	0.633	1.030	1.139	1.071	2.182	2.607
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.806	0.659	0.629	0.530	1.126	1.208	1.159	1.073	0.733	0.777	0.657	0.686
B-splines	27	27	0.876	0.698	0.690	0.558	0.923	1.347	1.011	1.231	0.907	0.748	0.576	0.787
Partitioning	27	27	0.903	1.476	0.709	1.150	0.831	1.189	0.850	1.833	0.732	3.762	0.837	4.304
Feasible Estimation														
Local Polynomial	0.26	0.26	0.740	0.602	0.581	0.484	1.223	1.214	1.274	1.180	0.582	0.684	0.614	0.530
B-splines	8	1	0.906	0.828	0.700	0.659	0.707	0.846	0.995	0.997	0.842	0.927	0.513	0.620
Partitioning	8	1	0.902	0.822	0.696	0.650	1.021	1.082	1.152	1.169	0.963	0.884	0.809	0.864

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.72: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators
$d = 3$, $n = 1000$, $\sigma^2 = 4$, $X_{i,\ell} \sim \beta(2,2)$, Quantile Cells

	Tuning Parameter		Root Integrated MSE		Ingetrated MAE		Point Estimation RMSE (0.5,0.5,0.5)		(0.1,0.5,0.5)		(0.1,0.1,0.5)		(0.1,0.1,0.1)	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Model 3.1														
Infeasible Estimation														
Local Polynomial	0.9	0.9	0.366	0.387	0.305	0.319	0.063	0.106	0.137	0.262	0.194	0.360	0.670	0.736
B-splines	1	1	0.370	0.428	0.307	0.344	0.063	0.123	0.132	0.344	0.276	0.946	0.705	1.153
Partitioning	1	1	0.124	0.281	0.096	0.204	0.062	0.121	0.131	0.302	0.170	0.561	0.211	0.935
Feasible Estimation														
Local Polynomial	0.37	0.27	0.505	0.521	0.410	0.422	0.073	0.117	0.232	0.290	0.692	0.721	1.313	1.332
B-splines	2	1	0.374	0.428	0.310	0.344	0.102	0.123	0.177	0.344	0.317	0.946	0.722	1.153
Partitioning	2	1	0.180	0.281	0.122	0.204	0.206	0.121	0.240	0.302	0.262	0.561	0.299	0.935
Model 3.2														
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.441	0.479	0.334	0.363	0.368	0.380	0.486	0.476	0.972	0.850	0.985	0.870
B-splines	27	8	0.437	0.475	0.339	0.359	0.367	0.391	0.404	0.549	0.729	1.320	0.773	1.338
Partitioning	27	8	0.680	0.807	0.529	0.606	0.367	2.036	0.713	2.585	0.992	2.923	1.158	3.643
Feasible Estimation														
Local Polynomial	0.33	0.25	0.379	0.400	0.287	0.305	0.380	0.388	0.379	0.405	0.365	0.405	0.364	0.392
B-splines	7	6	0.401	0.463	0.305	0.350	0.385	0.379	0.390	0.515	0.509	1.219	0.568	1.243
Partitioning	7	6	0.453	0.705	0.350	0.516	0.614	1.664	0.649	2.150	0.707	2.384	1.008	3.120
Model 3.3														
Infeasible Estimation														
Local Polynomial	0.38	0.39	0.178	0.257	0.124	0.183	0.078	0.133	0.243	0.326	0.504	0.629	0.504	0.629
B-splines	1	1	0.137	0.257	0.100	0.179	0.064	0.122	0.131	0.345	0.273	0.941	0.273	0.941
Partitioning	1	1	0.137	0.283	0.103	0.205	0.064	0.122	0.131	0.304	0.175	0.561	0.222	0.968
Feasible Estimation														
Local Polynomial	0.33	0.25	0.131	0.185	0.091	0.131	0.074	0.117	0.240	0.289	0.240	0.289	0.240	0.289
B-splines	6	3	0.183	0.274	0.133	0.190	0.183	0.154	0.283	0.373	0.439	1.048	0.439	1.048
Partitioning	6	3	0.318	0.469	0.232	0.302	0.437	0.961	0.481	1.279	0.489	1.430	0.545	1.991
Model 3.4														
Infeasible Estimation														
Local Polynomial	0.12	0.9	0.565	0.574	0.318	0.228	0.224	0.112	0.555	0.334	2.406	0.476	4.487	3.832
B-splines	64	27	0.616	0.595	0.325	0.336	0.399	0.187	0.583	0.541	1.381	2.687	4.054	4.659
Partitioning	64	27	1.063	1.471	0.817	1.143	2.389	1.232	2.228	4.593	2.056	9.670	3.085	20.072
Feasible Estimation														
Local Polynomial	0.31	0.25	0.569	0.569	0.194	0.223	0.075	0.117	0.335	0.382	0.335	0.382	3.799	3.799
B-splines	8	5	0.582	0.580	0.242	0.289	0.212	0.185	0.351	0.496	0.647	1.969	3.854	4.276
Partitioning	8	5	0.607	0.729	0.365	0.482	0.636	1.632	0.581	2.031	0.744	2.258	3.206	3.824
Model 3.5														
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.736	0.651	0.572	0.523	1.086	1.052	0.839	0.965	1.023	0.901	0.701	0.971
B-splines	27	27	0.803	0.709	0.619	0.562	0.955	1.178	0.713	0.995	1.153	2.078	0.781	1.934
Partitioning	27	27	0.866	1.476	0.669	1.148	0.632	1.235	0.764	4.568	1.063	9.675	1.154	20.204
Feasible Estimation														
Local Polynomial	0.27	0.25	0.712	0.624	0.559	0.501	1.201	1.179	1.002	1.176	0.881	0.736	0.390	0.550
B-splines	8	7	0.790	0.719	0.603	0.572	0.787	1.155	0.662	1.006	1.005	1.501	0.582	1.433
Partitioning	8	7	0.779	0.830	0.585	0.632	1.013	2.026	0.888	2.453	1.217	2.640	1.135	3.432

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.